Magnetism of the spin-1 tetramer compound $A_2Ni_2Mo_3O_{12}$ (A = Rb or K)

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We measured the temperature dependence of the magnetic susceptibility $\chi(T)$ and the specific heat C(T) and the magnetic-field dependence of the magnetization M(H) of $A_2Ni_2Mo_3O_{12}$ (A = Rb or K) powder. We consider that the probable spin model is an interacting spin-1 antiferromagnetic tetramer model. We evaluated values of the intratetramer interactions as $J_1 = 9$ K and $J_2 = 18$ K, and the effective intertetramer interaction as $J_{eff} = 4$ K for Rb₂Ni₂Mo₃O₁₂. The susceptibility and magnetization at 1.3 K of K₂Ni₂Mo₃O₁₂ are very close to those of Rb₂Ni₂Mo₃O₁₂. We observed a phase transition to a magnetically ordered state in C(T)/T in magnetic fields above 3 T. The transition temperature increases with magnetic field. Probably, the ordered state appears around 1.8 K even in 0 T. The ordered state in 0 T, however, is not stable enough like an order in the vicinity of a quantum critical point. Longitudinal-mode magnetic excitations may be observable in single crystalline $A_2Ni_2Mo_3O_{12}$ (A = Rb or K).

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I. INTRODUCTION

Two types of magnetic excitations exist in a magnetically ordered state. They are gapless transverse-mode (Nambu-Goldstone mode) [1] and gapped longitudinal-mode (amplitude Higgs mode) [2-6] excitations corresponding to fluctuations in directions perpendicular and parallel to ordered moments, respectively. The transverse-mode (T-mode) excitations are well known as spin wave excitations. Investigations of the longitudinal-mode (L-mode) excitations are now in progress. The L-mode excitation has weak intensity and spontaneously decays into a pair of T-mode excitations [7,8]. However, it can be well-defined in the ordered state in the vicinity of the quantum critical point [9]. In interacting antiferromagnetic (AF) spin- $\frac{1}{2}$ dimer compounds TlCuCl₃ and KCuCl₃, the ground state (GS) is a spin-singlet state at atmospheric pressure and zero magnetic field. The L-mode excitations were actually observed in a pressure-induced magnetically ordered state of TlCuCl₃ and KCuCl₃ by inelastic neutron scattering (INS) experiments [10–12] and Raman scattering experiments [13,14], respectively, and in a magnetic-field-induced ordered state of TlCuCl₃ by Raman scattering experiments [15].

According to results of theoretical investigations, the Lmode excitations may be observed in an antiferromagnetically ordered state appearing on cooling at atmospheric pressure and zero magnetic field in interacting spin-cluster compounds [16]. A shrinkage of ordered magnetic moments by quantum fluctuations leads to a large intensity of the L-mode excitations. If the GS of the corresponding isolated spin cluster is a spin-singlet state, the shrinkage of ordered moments can be expected in an ordered state of the interacting spin-cluster compounds. In interacting spin clusters, the ordered state can appear under the condition that the value of Δ is comparable to or less than that of an effective intercluster interaction [16]. Here Δ is the energy difference (spin gap) between the singlet GS and lowest triplet states in the isolated cluster. The effective intercluster interaction is given by the sum of the products of the absolute value of each intercluster interaction $(|J_{int,i}|)$ and the corresponding number of interactions per spin (z_i) as $J_{eff} = \sum_i z_i |J_{int,i}|$. The effective intercluster interaction is usually smaller than dominant intracluster interactions. Therefore it is advantageous to the appearance of the ordered state that Δ is smaller than dominant intracluster interactions. In the AF spin dimer given by $JS_1 \cdot S_2$, the value of Δ/J is 1 irrespective of the spin value. It is rare that spin dimer compounds show a magnetically ordered state at atmospheric pressure and zero magnetic field. Examples are NH₄CuCl₃ (spin $\frac{1}{2}$) [17,18] and CrVMoO₇ (spin $\frac{3}{2}$) [19].

In a spin tetramer expressed by the following Hamiltonian with $J_1 > 0$ or $J_2 > 0$, the GS is a spin-singlet state [20]. The Hamiltonian is

$$\mathcal{H} = J_1 S_2 \cdot S_3 + J_2 (S_1 \cdot S_2 + S_3 \cdot S_4). \tag{1}$$

 Δ/J_1 can be sufficiently small [21–24]. Interacting spin tetramers are advantageous to the appearance of the ordered state. Several spin tetramer compounds having a magnetically ordered state have been reported. Table I shows values of J_1, J_2, Δ , and T_N of the spin tetramer compounds. Magnetic excitations in Cu₂¹¹⁴Cd¹¹B₂O₆ were studied by INS experiments on its powder [23]. The results suggest the existence of the L-mode excitations. It is important to investigate the L-mode excitations in the known spin tetramer compounds using single crystals. It is also important to find more spin tetramer compounds having an antiferromagnetically ordered state. We can expect spin-1 tetramers in $A_2Ni_2Mo_3O_{12}$ (A = Rb or K) from its crystal structure [27,28]. We report magnetism of these compounds.

II. EXPECTED SPIN SYSTEM

The two compounds $A_2 Ni_2 Mo_3 O_{12}$ (A = Rb or K) are isostructural. The space group is $P2_1/c$ (No. 14). The lattice constants are a = 6.996, b = 9.186, c = 19.895 Å, and

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TABLE I. Values of the exchange interaction parameters, the spin gap of the isolated tetramers, and the AF transition temperature in the spin tetramer compounds. In Cu₂Fe₂Ge₄O₁₃, Cu²⁺ and Fe³⁺ ions have spins 1/2 and 5/2, respectively. The spin tetramers Fe-Cu-Cu-Fe are formed. In Rb₂Ni₂Mo₃O₁₂, T_N in 0 T was estimated from the extrapolation of $T_N(H)$.

	spin	<i>J</i> ₁ (K)	<i>J</i> ₂ (K)	Δ (K)	$T_{\rm N}$	Ref.
$\overline{Cu_2CdB_2O_6}$	$\frac{1}{2}$	317	-162	19	9.8	[23]
CuInVO ₅	$\frac{\tilde{1}}{2}$	240	-142	17	2.7	[24]
SeCuO ₃	$\frac{\tilde{1}}{2}$	225	160	84	8	[25]
Cu ₂ Fe ₂ Ge ₄ O ₁₃	$\frac{1}{2}, \frac{5}{2}$	255	26.7	1.3	39	[16,26]
$Rb_2Ni_2Mo_3O_{12}\\$	1	9	18	12	1.8(2)	this work

 $\beta = 108.71^{\circ}$ in Rb₂Ni₂Mo₃O₁₂ [27]. They are a = 6.952, b = 8.910, c = 19.733 Å, and $\beta = 108.06^{\circ}$ in K₂Ni₂Mo₃O₁₂ [28]. The Ni²⁺ ions (3 d^8) have localized spin-1. The positions of the Ni ions and the O ions connected to the Ni ions are shown schematically in Fig. 1(a). Two crystallographic Ni sites (Ni1 and Ni2) exist. Red and blue bars indicate two types of short Ni-Ni pairs. We show the Ni-Ni distances and Ni-O-Ni angles in Table II. If dominant exchange interactions exist in the Ni-Ni pairs, spin tetramers given by Eq. (1) are formed. The other Ni-Ni lengths are 4.999 (4.961) Å or greater in Rb₂Ni₂Mo₃O₁₂ (K₂Ni₂Mo₃O₁₂). Crystal structures depend on *A* and *M* in compounds expressed as A_2M_2 Mo₃O₁₂ (*A* = alkali metal and M = 3d metal). There are several types of spin systems. For example, the spin system is a frustrated spin- $\frac{1}{2}$ chain in A_2 Cu₂Mo₃O₁₂ (*A* = Rb or Cs) [29,30].

Figure 1(b) shows NiO₆ octahedra. Table III shows Ni-O lengths and O-Ni-O angles in the octahedra. The Ni1O₆ and Ni2O₆ octahedra are similar to each other. The symmetries of crystal fields affecting the Ni²⁺ ions are not so far from cubic. We expect that the single-ion anisotropy is small.

III. EXPERIMENTAL AND CALCULATION METHODS

Crystalline $A_2Ni_2Mo_3O_{12}$ (A = Rb or K) powder was synthesized by a solid-state reaction. Starting materials are Rb₂CO₃ (purity 99 %), K₂CO₃ (purity 99.9 %), NiO (purity 99.97 %), and MoO₃ (purity 99.99 %) powder. Stoichiometric mixtures of powder were sintered at 853 and 923 K in air for 150 h with intermediate grindings for the Rb and K compounds, respectively. We measured x-ray powder diffraction patterns at room temperature using an x-ray diffractometer (RINT-TTR III; Rigaku). We confirmed that each sample was a nearly single phase of $A_2Ni_2Mo_3O_{12}$ (A = Rb or K).

We measured the magnetization in magnetic fields of up to 5 T using a superconducting quantum interference device magnetometer magnetic property measurement system (Quantum Design). High-field magnetization measurements were conducted using an induction method with a multilayer pulsed field magnet installed at the Institute for Solid State Physics (ISSP), the University of Tokyo. We used a physical property measurement system (Quantum Design) for the specific heat measurements. We measured the specific heat in magnetic fields of not only $Rb_2Ni_2Mo_3O_{12}$ but also CuInVO₅ and CrVMoO₇ for comparison.

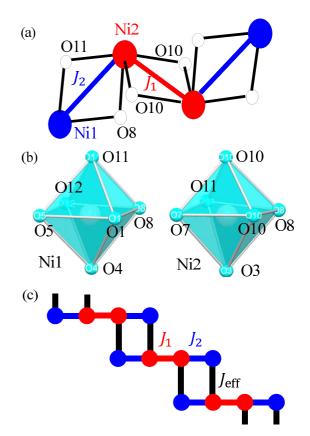


FIG. 1. (a) Schematic drawing of positions of Ni²⁺ ions having spin-1 and O²⁻ ions connected to Ni²⁺ ions in A_2 Ni₂Mo₃O₁₂ (A =Rb or K) [27,28]. Blue, red, and white circles indicate Ni1, Ni2, and O sites, respectively. Red and blue bars indicate two types of short Ni-Ni pairs. We define J_1 and J_2 as the exchange interaction parameters for the Ni2-Ni2 and Ni1-Ni2 pairs, respectively. The J_1 and J_2 interactions form a spin-1 tetramer. (b) NiO₆ octahedra of Ni1 (left) and Ni2 (right). (c) Interacting spin-1 tetramer model used to calculate magnetization using a mean-field theory based on the tetramer unit (tetramer mean-field theory). We adopted the interaction in the third shortest Ni-Ni pair (4.999 Å) as the effective intertetramer interaction (J_{eff}).

We obtained eigenenergies of isolated spin-1 tetramers using an exact diagonalization method. We calculated the temperature dependence of the magnetic susceptibility $\chi(T)$ and the specific heat C(T), and the magnetic-field dependence of the magnetization M(H) using the eigenenergies.

We calculated M(H) for the model shown in Fig. 1(c) using a mean-field theory based on the tetramer unit (tetramer

TABLE II. Ni-Ni lengths and Ni-O-Ni angles in the two types of short Ni-Ni pairs in A_2 Ni₂Mo₃O₁₂ (A = Rb or K).

		Rb Ni-Ni length (Å)	Ni-O-Ni angle (deg.)	K Ni-Ni length (Å)	Ni-O-Ni angle (deg.)
$J_1 \ J_2$	Ni2-Ni2 Ni1-Ni2	3.174 3.081	97.48 (O10) 96.31 (O8) 93.64 (O11)	3.128 3.089	96.24 (O10) 97.89 (O8) 93.96 (O11)

TABLE III. Ni-O lengths and O-Ni-O angles in NiO octahedra in Rb₂Ni₂Mo₃O₁₂. The deviation is defined as the difference between each Ni-O length and the average Ni-O length [$L_{ave,i}$ (i = 1,2)] divided by $L_{ave,i}$. The values of $L_{ave,i}$ are 2.080 and 2.062 Å for Ni1-O and Ni2-O, respectively. The maximum deviation in the Ni-O lengths from the average is 0.045. The minimum and maximum O-Ni-O angles are 81.88° and 101.18°, respectively.

	Å	deviation		Å	deviation
Ni1-O4	2.024	0.027	Ni2-07	2.023	0.019
01	2.035	0.022	O3	2.032	0.014
05	2.058	0.011	08	2.044	0.009
08	2.092	0.006	O11	2.050	0.006
012	2.098	0.009	O10(2)	2.077	0.007
011	2.174	0.045	O10(4)	2.145	0.040
	degrees			degrees	
011-Ni1-O1	90.58		O10(4)-Ni2-O7	84.43	
05	92.02		08	88.76	
O8	82.48		O10(2)	82.52	
012	81.88		O11	83.03	
O4-Ni1-O1	92.04		O3-Ni2-O7	95.17	
05	93.38		08	91.67	
O8	91.92		O10(2)	101.18	
012	95.31		O11	93.33	
O12-Ni1-O5	88.43		O10(2)-Ni2-O7	92.56	
O8	86.99		08	85.83	
O1-Ni1-O5	93.49		O11-Ni2-O7	93.09	
O8	90.42		08	86.78	
O11-Ni1-O4	173.84		O10(4)-Ni2-O3	176.30	
012-Ni1-O1	172.28		O10(2)-Ni2-O11	163.89	
08-Ni1-O5	173.29		08-Ni2-07	173.16	

mean-field theory). Finite magnetic moments were initially assumed on the Ni sites in the tetramer. The mean-field Hamiltonian was then expressed by a 81×81 matrix form under consideration of the external magnetic field and the molecular field from neighboring tetramers. The eigenstates of the mean-field Hamiltonian were used to calculate the expectation value of the ordered moments on the Ni sites. We continued this procedure until the values of the magnetic moments converged. We finally obtained a self-consistently determined solution for M(H).

IV. RESULTS AND DISCUSSION

Figure 2 shows the temperature T dependence of the magnetic susceptibility $\chi(T)$ of $A_2 Ni_2 Mo_3 O_{12}$ (A = Rb or K) in a magnetic field of H = 0.01 T. The susceptibilities of the two compounds are very close to each other. A broad maximum can be seen around 16 K, indicating a low-dimensional AF spin system. The susceptibility decreases rapidly at low T. However, $\chi(T)$ does not seem to approach a small value at 0 K expected for a spin singlet GS with a spin gap [31-33]. Figure 3 shows the magnetic field H dependence of the magnetization M(H) of Rb₂Ni₂Mo₃O₁₂ at 2 K. The magnetization has the finite slope even around 0 T, indicating that the GS is magnetic. The red circles in the inset of Fig. 4(a) show the specific heat divided by T [C(T)/T] of Rb₂Ni₂Mo₃O₁₂ in 0 T. A broad maximum can be seen around 5 K, indicating a low-dimensional AF spin system. C(T)/T seems to approach zero at 0 K, indicating no T-linear term. A spin liquid state cannot be expected [34]. We will describe later the results in finite magnetic fields.

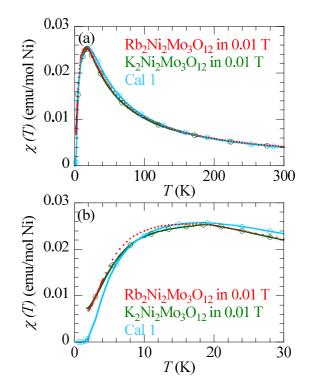


FIG. 2. Temperature *T* dependence of the magnetic susceptibility $\chi(T)$ of Rb₂Ni₂Mo₃O₁₂ (red circles) and K₂Ni₂Mo₃O₁₂ (green line) in a magnetic field of H = 0.01 T below 300 K (a) and 30 K (b). The light-blue line indicates $\chi(T)$ calculated for the isolated spin-1 tetramer with $J_1 = 9$ K and $J_2 = 18$ K.

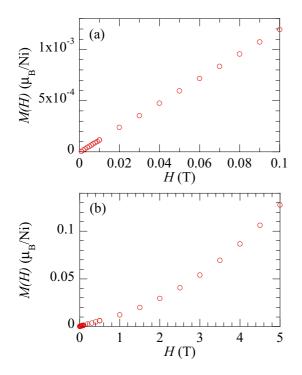


FIG. 3. Magnetic field H dependence of the magnetization M(H) of Rb₂Ni₂Mo₃O₁₂ at 2 K below 0.1 T (a) and 5 T (b).

The red lines in Fig. 5 show M(H) of Rb₂Ni₂Mo₃O₁₂. The magnetization at 1.3 K increases monotonically and is saturated around 45 T. The *g* value was evaluated to be 2.24 from the saturated magnetization. The saturation is also seen at 4.2 K and is smeared above 10 K. As shown in the inset of Fig. 5, the magnetizations of the two compounds at 1.3 K are very close to each other.

We compared experimental $\chi(T)$ and M(H) at 1.3 K with those calculated for the isolated spin-1 tetramer model expressed in Eq. (1). The experimental results are close to the calculated ones with $J_1 = 9$ K and $J_2 = 18$ K indicated by the light-blue lines in Figs. 2 and 5. The light-blue line in the inset of Fig. 4(a) indicates C(T)/T calculated for the same model. We cannot evaluate precisely the magnetic specific heat of Rb₂Ni₂Mo₃O₁₂. Although the red circles in the inset of Fig. 4(a) show the total C(T)/T including lattice specific heat, the experimental and calculated results are similar to each other.

The experimental M(H) at 1.3 K increases monotonically up to the saturation, whereas the calculated M(H) at 1.3 K shows quantum magnetization plateaus. According to the results in CuInVO₅ [24] and CrVMoO₇ [19], the discrepancy between the experimental and calculated M(H) is probably caused by intertetramer interactions. It is difficult, however, to determine which intertetramer interactions are effective. We adopted the interaction in the third shortest Ni-Ni pair (4.999 Å) as the effective intertetramer interaction (J_{eff}). Figure 1(c) shows schematically the interacting spin-1 tetramer model.

The blue lines in Fig. 5 indicate M(H) calculated for the interacting spin-1 tetramer model using the tetramer mean-field theory. The values of the exchange interactions are $J_1 = 9$ K, $J_2 = 18$ K, and $J_{\text{eff}} = 4$ K. The experimental and calculated M(H) at 1.3 K are in agreement with each other.

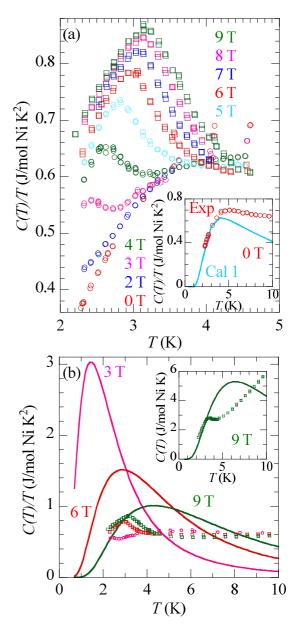


FIG. 4. (a) Temperature *T* dependence of the specific heat divided by *T* [*C*(*T*)/*T*] of Rb₂Ni₂Mo₃O₁₂ in various magnetic fields. The inset shows *C*(*T*)/*T* in zero magnetic field below 10 K. The light-blue line indicates *C*(*T*)/*T* calculated for the isolated spin-1 tetramer with $J_1 = 9$ K and $J_2 = 18$ K. (b) *C*(*T*)/*T* of Rb₂Ni₂Mo₃O₁₂ in 3, 6, and 9 T indicated by pink circles, red squares, and green squares, respectively. Lines indicate calculated Schottky specific heat of *g* = 2. The inset shows *C*(*T*) of Rb₂Ni₂Mo₃O₁₂ (squares) and the Schottky specific heat (line) in 9 T.

The calculated M(H), however, is larger than the experimental M(H) in high fields at 10 K and higher Ts. As J_1 or J_2 increases, the calculated M(H) at high T approaches the experimental M(H), whereas the discrepancy between the experimental and calculated results of M(H) at low T and $\chi(T)$ becomes apparent. We also calculated M(H) for the interacting spin-1 tetramer model including the D term of the single-ion anisotropy. Here, the Hamiltonian of the single-ion anisotropy is expressed as $\mathcal{H}_s = D(S^z)^2$. The calculated M(H)

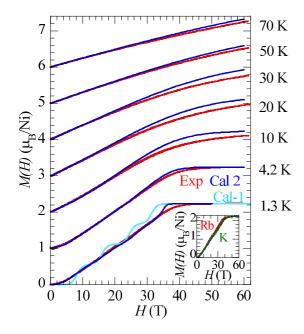


FIG. 5. Magnetic field *H* dependence of the magnetization M(H) of Rb₂Ni₂Mo₃O₁₂ (red lines) at various temperatures. Light-blue and blue lines indicate M(H) calculated for the isolated spin-1 tetramer model labeled by "cal 1" and for the interacting spin-1 tetramer model in Fig. 1(c) labeled by "cal 2," respectively. The values of the parameters are $J_1 = 9$ K, $J_2 = 18$ K, $J_{eff} = 4$ K, and g = 2.44. Vertical positions are shifted by 1 μ_B/Ni per line. The inset shows M(H) at 1.3 K of Rb₂Ni₂Mo₃O₁₂ (red) and K₂Ni₂Mo₃O₁₂ (green).

depends on the *D* value strongly and weakly at low and high *T*, respectively. The present discrepancy between the experimental and calculated M(H) could not be reduced by the introduction of the *D* term.

We can see 0, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ magnetization plateaus in the calculated line of the isolated spin-1 tetramer model at 1.3 K. The $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ magnetization-plateau phases are polarized paramagnetic phases in which $S_z^{T} = -1, -2, \text{ and } -3$, respectively. Here, S_z^{T} represents the *z* value of the total spin of the four S = 1 spins. In interacting spin tetramers, an ordered phase can appear in a magnetic-field range where M(H)increases. As described later, the specific-heat results indicate a magnetically ordered phase at low *T* in Rb₂Ni₂Mo₃O₁₂. No magnetization plateau in experimental lines indicates a single magnetically ordered phase is formed until the saturation of the magnetization [19].

Superexchange interactions are probably ferromagnetic in the two types of short Ni-Ni pairs because of the Ni-O-Ni angles. Antiferromagnetic direct exchange interactions are also expected because of the short Ni-Ni lengths. We infer that the summation of the ferromagnetic superexchange interactions and AF direct exchange interaction generates the small AF J_1 and J_2 interactions.

Figure 4(a) shows C(T)/T of Rb₂Ni₂Mo₃O₁₂ in various magnetic fields. We can see a peak above 3 T. The peak is not caused by isolated Ni²⁺ spins of impurities showing the Schottky specific heat. Figure 4(b) shows C(T)/T of Rb₂Ni₂Mo₃O₁₂ and that of the Schottky specific heat of g = 2. The Schottky specific heat has a stronger *H* dependence than

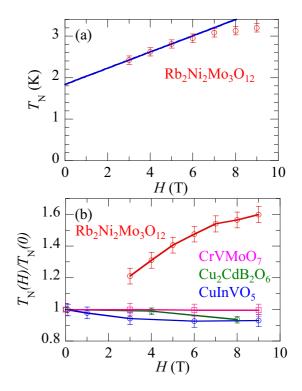


FIG. 6. (a) Magnetic field H dependence of the AF transition temperature $T_N(H)$ of Rb₂Ni₂Mo₃O₁₂. The line is a guide to eyes. (b) Magnetic field H dependence of $T_N(H)/T_N(0)$ of the spin tetramer compounds Rb₂Ni₂Mo₃O₁₂, Cu₂CdB₂O₆, and CuInVO₅. We used 2 K as $T_N(0)$ of Rb₂Ni₂Mo₃O₁₂ [the upper limit of estimated $T_N(0)$]. The results of the spin-3/2 dimer compound CrVMoO₇ are also shown as reference.

Rb₂Ni₂Mo₃O₁₂. The *H* dependence of the Schottky specific heat is stronger when *g* is larger. The inset of Fig. 4(b) shows C(T) in 9 T of Rb₂Ni₂Mo₃O₁₂ and the Schottky specific heat of g = 2. The peak positions are different from each other. The difference is larger when *g* is larger. Consequently, the peaks in C(T)/T of Rb₂Ni₂Mo₃O₁₂ indicate a phase transition to a magnetically ordered state.

Figure 6(a) shows the *H* dependence of the peak temperature $T_N(H)$ in C(T)/T of Rb₂Ni₂Mo₃O₁₂. The peak temperature increases with *H*. As suggested by the blue line, we infer that the ordered state appears around 1.8 K even in 0 T. This inference is consistent with the susceptibility and magnetization results indicating that the GS is magnetic.

Figure 6(b) shows the *H* dependence of $T_N(H)/T_N(0)$ of the spin tetramer compounds Rb₂Ni₂Mo₃O₁₂, Cu₂CdB₂O₆ [35], and CuInVO₅, and the spin-3/2 dimer compound CrVMoO₇. As *H* is increased, $T_N(H)/T_N(0)$ increases in Rb₂Ni₂Mo₃O₁₂, whereas $T_N(H)/T_N(0)$ is almost constant or decreases slightly in the other three compounds. The phase transition temperature increases with *H* in the spin-tetrahedra system Cu₂Te₂O₅Br₂ [36]. Investigations using chemical and hydrostatic pressure suggest the closeness of the system to a quantum critical point [37–40]. In Rb₂Ni₂Mo₃O₁₂, we speculate that the order in 0 T is not stable enough like an order in the vicinity of a quantum critical point [10–12] and that the order is stabilized by the application of magnetic fields.

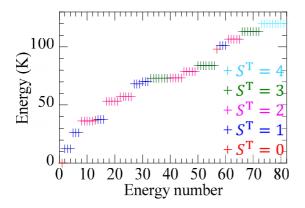


FIG. 7. Eigenenergies measured from the spin-singlet ground state in the isolated spin-1 tetramer with $J_1 = 9$ K and $J_2 = 18$ K.

Figure 7 shows the eigenenergies measured from the spinsinglet GS in the isolated spin-1 tetramer with $J_1 = 9$ K and $J_2 = 18$ K. The spin gap value (12 K) is smaller than the dominant J_2 value and is larger than the J_{eff} value (4 K). We consider that the order in 0 T is not stable enough because of the small J_{eff} value.

In future, we will make single crystals of $A_2Ni_2Mo_3O_{12}$ (A = Rb or K) and evaluate the single-ion anisotropy. We will determine the magnetic structure by neutron diffraction experiments. We will confirm the signs of J_1 and J_2 from the magnetic structure. We will consider which intertetramer interactions are effective to stabilize the magnetic structure. We will calculate $\chi(T)$ and M(H) of a more realistic model using quantum Monte Carlo techniques and reproduce the experimental results.

As described, the order of $Rb_2Ni_2Mo_3O_{12}$ in 0 T may be similar to an order in the vicinity of a quantum critical point. The L-mode magnetic excitations may be observable like in the pressure-induced or magnetic-field-induced magnetically ordered state of the interacting AF spin- $\frac{1}{2}$ dimer compounds TlCuCl₃ and KCuCl₃ [10–15]. We intend to perform inelastic neutron scattering and Raman scattering experiments to investigate L-mode magnetic excitations. In the weakly ordered spin- $\frac{1}{2}$ chain antiferromagnet Sr₂CuO₃, unusual magnetic excitations were recently observed by ESR experiments [41]. It is reported that the excitations can be attributed to the Nambu-Goldstone mode renormalized due to its interaction with the high-energy L-mode. We also pursue such unusual excitations in $A_2Ni_2Mo_3O_{12}$ (A = Rb or K).

V. CONCLUSION

We measured the temperature T dependence of the magnetic susceptibility $\chi(T)$ and the specific heat C(T) and the magnetic-field H dependence of the magnetization M(H) of Rb₂Ni₂Mo₃O₁₂ powder. A broad maximum appears around 16 K in $\chi(T)$. Although $\chi(T)$ decreases rapidly at low T, $\chi(T)$ does not seem to approach a small value at 0 K expected for a spin singlet GS with a spin gap. The low-field magnetization at 2 K has the finite slope even around 0 T. The high-field magnetization at 1.3 K increases monotonically without magnetization plateaus and is saturated around 45 T. The susceptibility and magnetization at 1.3 K of K2Ni2Mo3O12 are very close to those of Rb₂Ni₂Mo₃O₁₂. The isolated spin-1 antiferromagnetic tetramer model with $J_1 = 9$ K and $J_2 = 18$ K can closely reproduce the experimental susceptibility. We were able to explain the magnetization curves using the interacting spin-1 tetramer model with the effective intertetramer interaction $J_{\text{eff}} = 4$ K. In C(T)/T, we can see a peak above 3 T, indicating a phase transition to a magnetically ordered state. The transition temperature $T_{\rm N}(H)$ increases with H. From the *H* dependence of $T_N(H)$, probably, the ordered state appears around 1.8 K even in 0 T. The ordered state in 0 T, however, is not stable enough like an order in the vicinity of a quantum critical point. Longitudinal-mode magnetic excitations may be observable in single crystalline $A_2Ni_2Mo_3O_{12}$ $(A = \operatorname{Rb} \operatorname{or} K).$

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