

Thermal properties of the superconductor-quantum Hall interfaces

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An important route of engineering topological states and excitations is to combine superconductors (SC) with the quantum Hall (QH) effect, and over the past decade, significant progress has been made in this direction. While typical measurements of these states focus on electronic properties, little attention has been paid to the accompanying thermal responses. Here, we examine the thermal properties of the interface between a type-II superconducting electrodes and graphene in the QH regime. We use the thermal noise measurement to probe the local electron temperature of the superconductor next to the biased interface. Surprisingly, the measured temperature raise indicates that the superconductor provides a significant thermal conductivity, which is linear in temperature. This suggests electronic heat transport and may be unexpected, because the number of the quasiparticles in the superconductor should be exponentially suppressed. Instead, we attribute the measured electronic heat conductivity to the overlap of the normal states in the vortex cores.

Over the past decade, significant progress has been made in combining quantum Hall states with superconductors [1–7]. The experiments clearly demonstrate the coherent quasiparticle propagation in the proximitized chiral interfaces [3, 4, 6]. While these works focussed on the time-averaged transport, theory predicts that thermal properties and fluctuations should carry unique information, e.g. about the nature of the neutral excitations [8]. However, with a few exceptions [9], little experimental evidence exists about the thermal properties of the proximitized semiconductor interfaces.

Thermal properties of the nanoscale quantum materials emerged as an interesting novel area of research [10]. While such quantities as Nernst and Seebeck coefficients became relatively well known, thermal conductivity measurements remain less explored [11]. Recent studies demonstrate that measurements of mK temperatures and fW powers are feasible in appropriately designed QH samples via Johnson-Nyquist noise [12–15]. We use a similar noise setup to examine the thermal response of a hybrid device made of graphene contacted by type-II thin film superconducting electrodes. In the QH regime, we bias the device forming hot spots – regions where the Joule heating is deposited – at the interface with the contact, from where the heat can escape via several mechanisms. We then measure the noise carried by the QH edge states downstream of the superconductor to probe its local temperature.

We find that applying current on the scale of tens of nA can heat the interfacial region of the contact to ~ 1 K. As the magnetic field increases, the electron temperature gradually decreases until becoming comparable to the temperature of a similarly heated normal contact. We argue that this increase of the cooling efficiency is explained by the electronic heat conductivity of the superconductor, mediated by the normal states in the vortex cores [16]. This mechanism becomes more efficient as the distance between the vortices decreases with magnetic field. Further examination of the temperature response across a few μm -wide superconducting strip reveals

that indeed the heat spreads rather efficiently across superconductor. Eventually, at high magnetic field, the whole width of the strip becomes uniformly heated. The estimated thermal conductivity varies by almost two orders of magnitude over the studied range of magnetic field.

The schematic and the image of our sample are shown in Figure 1a. The device is made of encapsulated graphene contacted by normal metal (Cr/Au) and superconductor (MoRe) electrodes. Details of the device can be found in Ref. [17]. Throughout the measurement, we apply a field of several Tesla to induce the QH effect in graphene. The MoRe has a critical temperature $T_c \sim 10$ K and stays superconducting at least up to 12 T, the highest field in this measurement (Figure S1 in [18]). The width of the superconducting strip (light gray in Figure 1a) is $\sim 2 \mu\text{m}$, much longer than the MoRe coherence length ($\xi < 10$ nm). Therefore, in the first part of this paper the two graphene-superconductor interfaces will be considered separately.

The noise at the two interfaces is measured by two home-made cryogenic amplifiers (~ 10 dB gain), A and B, which are attached to the normal electrodes, E_A and E_B , at the top and bottom regions respectively (Figure 1a). The amplifiers are AC coupled to the sample electrodes via 10 nF blocking capacitors and LC resonators centered around 1.5 MHz. The outputs of the cryogenic amplifiers are then fed to room temperature amplifiers (46 dB gain) and digitized.

We start by exploring the thermalization of the edge state in contact with the superconductor. We work with the bottom region of the sample (Figure 1a), and measure the spectral density S_B of amplifier B. Depending on the field direction, the QH edge channels will arrive at the amplifier contacts either from the superconducting strip (positive field) or from the bottom normal contact (negative field). To enable a direct comparison, the length of the bottom normal metal and the bottom superconducting interfaces are designed to be the same ($1 \mu\text{m}$), and both contacts are cold grounded.

Figure 1b plots the zero-bias S_B measured vs. bath tem-

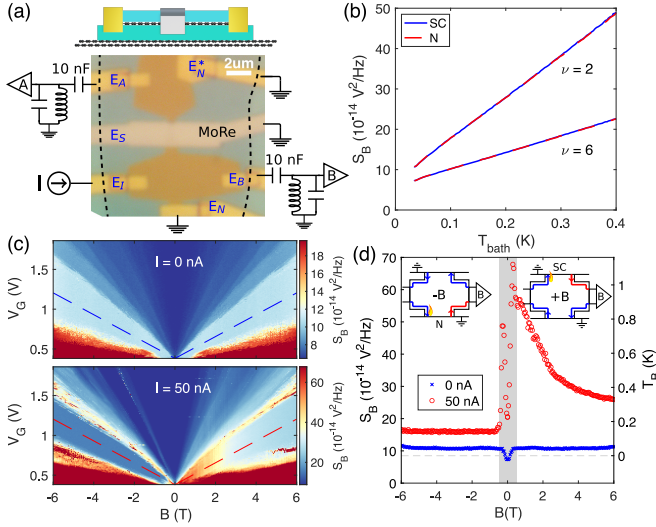


FIG. 1. **Excess noise of superconducting and normal contacts.** (a) Schematics of the device and measurement configuration. Normal contacts (Cr/Au) are yellow and the superconducting contact (MoRe) is gray. The cross section (top) shows the hBN/graphene/hBN stack sitting on a graphite gate (outlined by black dashed lines in the optical image). The superconductor, the bottom and top-right normal contacts are cold grounded. Current bias I is injected at the bottom left normal contact. In the remainder of the figure, only the bottom region of graphene is measured. (b) Zero-bias S_B of the normal metal (red, $B = -3 \text{ T}$) and superconductor (blue, $B = 3 \text{ T}$) plotted vs. the bath temperature T_{bath} . The top and bottom curves are obtained on the $\nu = 2$ and 6 plateaus. (c) Noise power S_B measured by amplifier B (after the amplification chain) plotted vs. B and V_G at zero bias (top) and 50 nA (bottom). (d) Cuts of the maps in (c) measured along the dashed lines (middle of $\nu = 2$ plateau). Insets: schematics of the edge state direction, and locations of the relevant hot spots and hot edges for negative and positive fields.

perature T_{bath} for both $\nu = 2$ and 6 at $|B| = 3 \text{ T}$. The result for the superconductor and the normal contact are practically indistinguishable. The noise is linear vs. T_{bath} , allowing us to calibrate the gain of the amplification chain and convert S_B to temperature [18]. Next, we measure S_B at the base temperature $T_{\text{bath}} = T_0$ as a function of V_G and B (top panel of Figure 1c). The noise map is nearly symmetric with respect to the field direction, and the noise stays constant on the plateaus, indicating that both the superconducting and the reference normal contact stay thermalized at the base temperature of the sample, $T_0 = 35 \text{ mK}$, throughout the full range of magnetic field. In the following, we disregard a narrow range of fields $|B| < 1 \text{ T}$, where the conductance of the sample is no longer quantized (see Figure S2 in [18]).

We next apply a current bias of $I = 50 \text{ nA}$ to the bottom-left contact and plot the resulting $S_B(V_G, B)$ in the bottom panel of Figure 1c. At negative B , when we locally heat the bottom normal contact, the noise on the $\nu = 2$ plateau increases compared to zero bias. This indicates that the normal electrode is locally heated by the applied power, and its thermal noise is then detected by the amplifier B located downstream. Note that in this case, the excess noise stays con-

stant in B . At positive B , when the superconductor is locally heated, the excess noise on the $\nu = 2$ plateau stays constant in V_G , but strongly dependent on magnetic field.

In Figure 1d we plot the noise measured along the dashed lines corresponding to the $\nu = 2$ plateaus in Figure 1c. This noise (left axis) is converted to the temperature (right axis) via the calibration developed in Figure 1b. At $I = 0$, the noise on the $\nu = 2$ plateau stays constant in B , except for the small vicinity of zero field. At finite bias, the elevated noise stays constant at negative fields, indicating that the normal contact is heated to about 100 mK independent of the field. At positive field corresponding to biasing the superconductor, we observe a strong enhancement of the noise followed by a gradual decrease.

In principle, the increased noise at the biased superconductor-QH interface could have originated from the shot noise [19] (see [18] for a discussion of other noise sources). Namely, in the case of an ideal superconductor, an incoming electron would have to undergo either a normal or Andreev reflection. Strong current fluctuations could then be expected, corresponding to the electrons or holes being emitted downstream (toward the amplifier). However, the probabilities of the normal and Andreev processes strongly depend on the gate voltage [4], while experimentally, the noise at a fixed field stays flat across the plateaus (Figure 1c). (Very small variations of the noise, on the scale of 1%, have been observed as a function of gate voltage.) Furthermore, our previous studies of the superconductor-QH interfaces, including this very sample [17], indicate that most of the electrons arriving at the superconducting contact are absorbed, likely by the Caroli-de Gennes-Matricorn (CdGM) states of the superconducting vortices [20]. We argue that as a result, the interfacial region of the superconductor is locally heated, and its thermal noise is carried downstream to the amplifier, similar to the case of the normal contact. In [18], we independently verified the temperature rise at the superconductor-QH interface by measuring the suppression of the non-local resistances.

In the following, we convert the excess noise to the local electron temperature using the calibration of Figure 1b. (Notice that additional factor of 2 has to be applied in this conversion to account for the fact that the amplifier contact stays cold [12, 18].) In Figure 2a, we plot the temperature of the superconductor next to the bottom interface $T(I)$, as the field is stepped from $B = 1$ to 12 T. The gate voltage is adjusted to stay in the middle of the $\nu = 2$ plateau. At $B = 1 \text{ T}$, the temperature reaches $\sim 1 \text{ K}$ at 100 nA. Even at a 10 nA bias (corresponding to a voltage drop of $\sim 130 \mu\text{V}$), the electron temperature increases to about 300 mK. As B increases, T decreases precipitously, as show by the $T(B)$ graphs measured at fixed I in Figure 2b. At the highest field of 12 T, the temperature of the superconductor next to the interface becomes comparable to that of the reference normal contact, whose $T(I)$ is plotted in Figure 2a as a dashed line. Interestingly, at that point MoRe is still superconducting (Figure S1 in [18]).

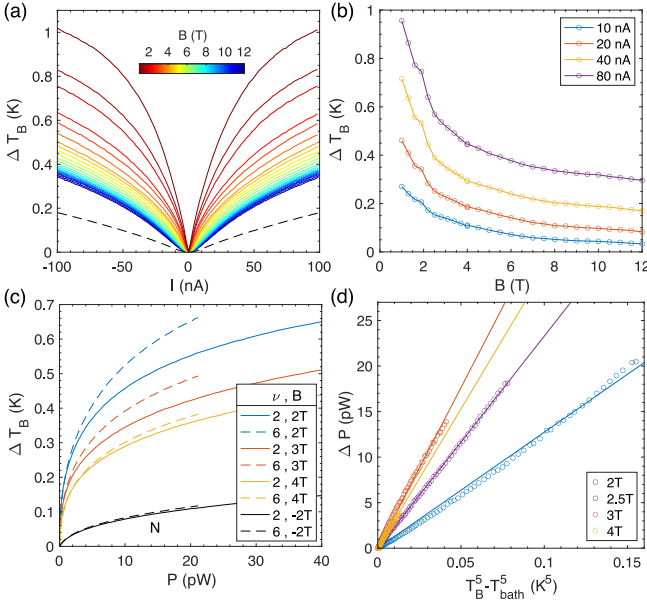


FIG. 2. **Heating of the bottom interface.** (a) Temperature increase ΔT_B plotted vs. bias current I for $\nu = 2$. The different curves corresponds to magnetic fields B is stepped by 0.1 T (1–4 T range) and 0.5 T (4 – 12 T range). (b) ΔT_B vs. B for $\nu = 2$ is plotted at constant bias currents of 10, 20, 40 and 80 nA, showing the gradual decay of temperature with field. Variations of temperature observed at small fields for all curves are likely caused by changes of the cooling pathways due to vortex rearrangements, see Figure S3 in [18]. (c) ΔT_B plotted vs. the Joule power P at $T_{\text{bath}} = 35$ mK. Different colors correspond to magnetic fields of 2, 3 & 4 T, while $\nu = 2$ and 6 are represented by the solid and dashed lines. The corresponding $\Delta T_B(P)$ curves of the normal contact are nearly independent on B and ν (black lines). (d) The difference in power between the $\nu = 2$ and 6 curves of panel (c) plotted against $T_B^5 - T_{\text{bath}}^5$. The linear dependence indicates that the difference can be attributed to the contribution of phonons.

The $T(I)$ curves in Figure 2a have a V-shape typical for hot electrons in a metal that are cooled via diffusion (Wiedemann-Franz mechanism) and the emission of phonons [12]. Namely, at low temperatures, the phonon emission is negligible, and the applied power $P \propto I^2$ is balanced by electronic cooling $\propto (T^2 - T_0^2)$, resulting in the roughly linear slope of the $T(I)$ curves visible in Figure 2a. At higher temperatures, the emission of phonons by hot electrons results in the sublinear $T(I)$ [21].

Next, we plot $T(P)$ – the temperature of the superconductor next to the interface as a function of applied power for three magnetic field values (2, 3 & 4 T) in Figure 2c. Here $P = I^2 R/2$, where the factor of 1/2 appears because the Joule heating is evenly distributed between two hot spots. For comparison, we also plot the $T(P)$ of the normal metal at 2 T, which are nearly independent of B .

The focus of this figure is to compare the behavior for the filling factors $\nu = 2$ (solid lines) and $\nu = 6$ (dashed lines). At a given field, the pairs of the $T(P)$ curves for $\nu = 2$ and 6 overlap at low power, but diverge at high power. Surpris-

ingly, the temperature at $\nu = 6$ is hotter, ruling out the more efficient cooling via QH edges. Moreover, the difference is noticeable at higher power, where the phonon emission is the dominant cooling mechanism. To prove this point, we plot the difference in power, ΔP , needed to reach the same T vs. $T^\delta - T_0^\delta$ and find a linear relation for $\delta = 5$ (Figure 2d). $\Delta P \propto (T^5 - T_0^5)$ is the typical cooling power of the hot electrons via phonon emission in diffusive metals [21].

While we cannot pinpoint the exact origin of the difference between the two filling factors, we attribute it to the difference in the size of the QH hot spot, which is determined by the length over which the QH edge channel is equilibrated with the contact. Notice that a similar difference is also observed for the normal contact (black lines) – $\nu = 2$ is slightly cooler than $\nu = 6$ – so this effect is not specific to the superconductor.

Finally, we use the data in Figure 2 to bring forward two arguments further supporting the attribution of the excess noise to the electron temperature. First, $T(P)$ curves for $\nu = 2$ and 6 nearly coincide at low power. However, a significantly enhanced shot noise could have been expected for $\nu = 6$ vs. $\nu = 2$, because a given power P would correspond to a $\sqrt{3}$ times higher bias current. Second, in principle we could still attempt to fit the $S(I)$ curves with the shot noise expression, $S \propto I \times \coth(eV/2k_B T_e)$, with $V = hI/\nu e^2$. The curvature at the minimum of the $S(I)$ curves would then correspond to the electrons' base temperature T_e treated as a fitting parameter. T_e would then depend on both ν and B , increasing with magnetic field to ~ 200 mK at 12 T. This result would both be physically unreasonable and contradict Figure 1d. We therefore rule out the shot noise as an explanation of the measured $S(I)$ curves.

To further analyze the purely electronic cooling, in Figure 3a we plot T^2 vs. applied power in the range of $P < 500$ pW and $T < 100$ mK – at which temperatures the phonon emission should be negligible [21]. The data points show a clear linear dependence, indicating that the superconductor is indeed cooled via the diffusion of hot electrons and the emission of phonons can be neglected, with a possible exception of the lowest fields ($B \lesssim 1.5$ T). We further extract the slope of the $P = \alpha(T_B^2 - T_0^2)$ dependence (Figure 3b). The coefficient α is presented in the quantized units of $\alpha_Q = \pi^2 k_B/6h$, corresponding to the heat flow in a single QH channel [12]. Even at the lowest field, the measured α greatly exceeds that of the quantum Hall. We therefore attribute it to the electronic heat conductivity of the superconductor, which appears to be linear in temperature, like in a normal metal.

At zero field, the electronic heat conductivity of a superconductor is strongly suppressed due to the gap in the single-particle spectrum. (Though violation of this dependence has been reported see e.g. [22].) However, at finite field, the CdGM states should behave like a normal metal, and the tunneling of the quasiparticles between the vortex cores can efficiently conduct heat. The resulting thermal conductivity is linear in temperature and rapidly grows with magnetic field due to the increasing overlap between the vortex cores [16].

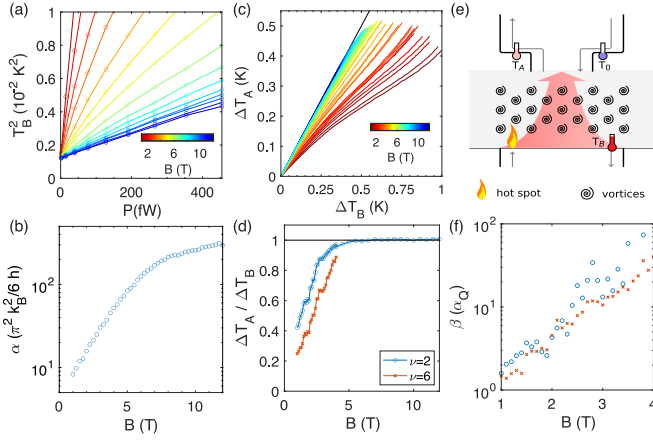


FIG. 3. Thermal conductivity of the superconducting film. (a) T_B^2 plotted vs. Joule heating power P from $B = 1$ to 11.5 T (shown in steps of 0.75 T). Linear dependence is clearly visible indicating electronic thermal conductivity. (b) The slope α extracted from the $T_B^2(P)$ curves in panel (a) as a function of B . α is represented in units of $\alpha_Q = \pi^2 k_B / 6h$, corresponding to a single QH channel. (c) ΔT_A plotted vs. ΔT_B on the $\nu = 2$ plateau from $B = 1$ to 12 T. The T_B data is the same as in Figure 2a. The black line is $\Delta T_A = \Delta T_B$. (d) $\Delta T_A / \Delta T_B$ ratio extracted from the lower range of temperatures in panel (c), plotted vs. B for $\nu = 2$ (blue circles) and 6 (red crosses). The quantity converge to unity at high magnetic field. (e) Schematics of the heat flow across the superconductor showing the heat balance at the top interface. (f) β , plotted in units of α_Q , represents the electronic thermal conductivity across the top interface (please see main text). $\beta(B)$ dependencies nearly overlap for $\nu = 2$ (blue circles) and $\nu = 6$ (red crosses).

This regime was tentatively reported in the seventies [23], and more recently explored in Ref. [24]. Closer to H_{c2} , the order parameter becomes nearly uniformly suppressed, which increases the density of quasiparticles [25]. In this regime, α is linearly approaching the value corresponding to the normal state of the metal [26]. Both regimes – the initial rapid rise followed by flattening close to H_{c2} – are clearly visible in Figure 3b, which may be the first such demonstration in thin film samples.

We are now interested to find out what happens once the heat spreads from the QH hot spot through the superconductor. To that end, we use amplifier A (Figure 1a) to study the temperature of the edge state which originates at the top superconducting interface, T_A . (We also reinstate subscript B to indicate the temperature measured by the bottom amplifier interface, T_B .) In Figure S4 of [18], we plot T_A vs. P while the power is still applied by biasing the bottom left contact, the same way as in Figure 2. Evidently, the heat spreads efficiently to the opposite side of the superconducting strip.

In Figure 3c we plot $\Delta T_A = T_A - T_0$ vs. ΔT_B . Experimentally, we find that ΔT_A scales almost linearly with ΔT_B . The robust nature of this relation, which extends over the range of temperature changes much larger than the base temperature, is presently not clear. We extract the $\Delta T_A / \Delta T_B$ slope at low power and plot it in Figure 3d as a function of B . Similar data

for several applied powers is presented in Figure S5 of [18]. Independent of P , the slope increases monotonically with B and saturates at 1 for $B \gtrsim 4$ T, directly confirming that the whole width of the superconducting strip gets uniformly hot.

To understand the behavior in the lower field range, $B \lesssim 4$ T, we note the difference between the plots of $\Delta T_A / \Delta T_B$ measured at $\nu = 2$ and $\nu = 6$ (lower curve in Figure 3d). We know that the superconductor is highly thermally conductive for the whole range of fields $B > 1$ T (Figure 3b), and the 2 vs. 6 additional quantum channels of graphene should not change the temperature of the metal itself. We therefore argue that T_A is the temperature of the QH edge running along the relatively short top interface, which has not fully equilibrated with the hot electrons in the superconductor.

We analyze the heat flow at the top superconducting interface as sketched in Figure 3e. Given the high heat conductivity of the superconductor (Figure 3b), we approximate the electron temperature of the superconducting metal close to the top interface as T_B . The edge channels arrive at the interface with temperature T_0 and acquire heat from the superconductor, eventually reaching a temperature of T_A . This temperature should satisfy the heat balance equation, $\beta(T_B^2 - T_A^2) = \nu \alpha_Q (T_A^2 - T_0^2)$, where β is proposed to represent the effective electronic heat conductivity across the top interface. Using this expression, we extract β by fitting $T_B^2 - T_A^2$ vs. $T_A^2 - T_0^2$ and find that the values nearly coincide for the two filling factors $\nu = 2$ and 6 (Figure 3f). We note that β is different from α in Figure 3b, which characterizes the electronic heat conductivity of the superconductor itself. In contrast, β characterizes the heat conductivity between the metal and the edge state at the top interface.

Summarizing our results, we find that the dissipation at the hot spot results in a substantial overheating of the superconducting metal close to the SC-QH interfaces, which is particularly noticeable at low magnetic fields. Carefully engineered heat sinks would be necessary for quantum applications of such systems e.g. for quantum information processing. Increasing the magnetic field rapidly thermalizes the superconductor, resulting in a change of the thermal conductivity by more than one order of magnitude (Figure 3b). By $\sim H_{c2}/2$, the heat dissipation by the superconductor becomes comparable to that of the normal electrode. Finally, our observations are unfavorable for measuring the heat conductivity of the proximitized QH edge states along the SC interface – unfortunately, they would be thermally shunted by the thermal conductivity of the superconductor.

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