

Graphene as a Tunable Nonradiative Bath for Moiré Excitons

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Abstract

A minimal theory of nonradiative energy transfer from a two-dimensional (2D) moiré exciton to a nearby graphene layer is presented. From Fermi’s golden rule the transfer rate is the overlap of the exciton near-field spectrum with the dissipative density response of graphene, weighted by an exciton form factor, and it reproduces the established $\Gamma_{\text{ET}} \propto z^{-4}$ law in the point-dipole limit. A finite exciton size filters out the high-momentum near field once the spacer thickness approaches the transition-polarization radius R_X , so the distance dependence of the rate—and of the photoluminescence (PL) quenching—probes the exciton size. A low-momentum expansion shows that, relative to the calibrated point-dipole response of the same bath, this leading correction is set by R_X alone. In the ideal coherent-envelope limit the accompanying giant oscillator strength makes the rate non-monotonic in the exciton size, with a peak near $\ell_X \approx z$. Treating graphene as a gate-tunable bath, Pauli blocking suppresses the interband channel once $2|\mu_F|$ approaches $\hbar\omega$, partially restoring PL, and a full random-phase-approximation benchmark confirms the normalized interband distance dependence to within a few percent away from the threshold. Mapping the PL observables across the transition-metal dichalcogenide/hexagonal boron nitride/graphene parameter space, we find that a graphene gate acts not as a passive electrostatic element but as a tunable 2D electronic reservoir probed through exciton PL quenching.

I. INTRODUCTION

When an excited emitter is placed near a conducting or semimetallic system, it can relax by transferring its energy nonradiatively to electronic excitations of that system instead of emitting a photon. For graphene, Swathi and Sebastian showed using a tight-binding model and the Dirac-cone approximation that the resonance energy-transfer (ET) rate from a point dye molecule scales as z^{-4} with the emitter–graphene distance z [1, 2], much longer ranged than the R^{-6} Förster law of molecular fluorescence resonance energy transfer [3]. This distance scaling has been observed experimentally for individual quantum emitters and semiconductor nanostructures [4–6], and the relaxation pathway has been shown to be electrically controllable through the graphene Fermi level [7, 8]. On the theory side, the graphene fluorescence-quenching problem has been revisited with the full electronic

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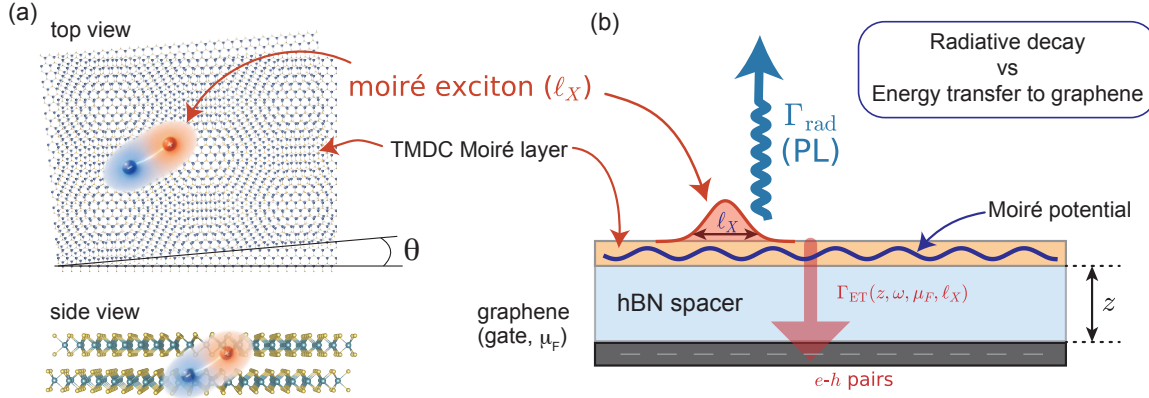


FIG. 1. (Color online) (a) Moiré exciton—a bound electron–hole pair of center-of-mass extent ℓ_X —in a transition-metal dichalcogenide (TMDC) heterobilayer twisted by angle θ . Top view: the exciton trapped in the moiré superlattice; side view: the electron and hole occupy adjacent layers, forming an interlayer exciton. (b) Device cross section: the exciton in the TMDC moiré layer, confined by the moiré potential, can decay radiatively, emitting photoluminescence (PL) at rate Γ_{rad} , or transfer its energy nonradiatively across a hexagonal boron nitride spacer of thickness z to electron–hole pairs in a graphene gate at Fermi level μ_F [$\Gamma_{\text{ET}}(z, \omega, \mu_F, \ell_X)$], the channel studied here.

response of graphene, clarifying the role of its loss function and of longitudinal and transverse plasmons [9, 10].

Two-dimensional (2D) semiconductors and their moiré superlattices provide a qualitatively richer setting for this physics. The optical response of monolayer and bilayer transition-metal dichalcogenides (TMDCs) is dominated by tightly bound excitons with binding energies of hundreds of meV [11, 12]. In TMDC heterobilayers, interlayer excitons can be trapped in the moiré potential, forming arrays of localized emitters whose spatial extent ℓ_X ranges from a few to tens of nanometers [13–18]; the theory and broader phenomenology of these moiré-trapped emitters are reviewed in Refs. [19–21], and they act as gate-tunable quantum-light sources [22]. These devices commonly include nearby graphene or graphite layers acting as gates or contacts, separated from the excitons by only a few nanometers of hexagonal boron nitride (hBN) [Fig. 1]. The same electrode used for electrostatic control therefore sits within energy-transfer range of the excitons.

TABLE I. Emitters near a 2D electronic bath, classified by the (amplitude) exciton form factor $F_X(q)$. The rate (5) carries $|F_X(q)|^2$. The tabulated Gaussian form factors correspond to the coherent model, for which a spatially coherent exciton additionally carries a giant oscillator strength $|D_X|^2 \propto \ell_X^2$. The fixed-dipole benchmark instead holds D_X fixed and replaces $|F_X|^2 = e^{-q^2 \ell_X^2}$ by $e^{-q^2 \ell_X^2/2}$.

emitter	$F_X(q)$	localization
point dipole / molecule	1	none (z^{-4} limit)
finite-coherence exciton	effective	weak, uncontrolled
disorder-localized	$e^{-q^2 \ell_{\text{dis}}^2/2}$	random
moiré exciton	$e^{-q^2 \ell_X^2/2}$	tunable

The nonradiative mechanism itself is general. It is not specific to moiré excitons but operates for any emitter—a dye molecule, a quantum dot, a free or disorder-localized monolayer exciton—placed near a 2D electronic bath. What distinguishes these cases is the emitter’s spatial extent, which enters the transfer rate through a form factor $F_X(q)$ and, for a spatially coherent emitter, through its total transition dipole D_X (Table I). A point dipole has $F_X \rightarrow 1$ and recovers the Swathi–Sebastian result. A delocalized monolayer exciton has an effective F_X set by coherence, temperature, and disorder, none of them easily controlled. A moiré exciton is the cleanest realization. Its center-of-mass wave function is localized by the moiré potential [23], so its localization length ℓ_X is an *intrinsic and tunable* parameter—set by twist angle, moiré-potential depth, and registry. Proximity quenching thus becomes a controllable probe of exciton localization, and we take moiré excitons as the primary application of an otherwise general formalism.

In this work we reformulate graphene-induced fluorescence quenching for the modern context of moiré excitons. Related resonance-energy-transfer problems—graphene near quantum dots, defect-bound single-photon emitters, or moiré-trapped excitons—have been treated within the random-phase approximation (RPA), which evaluates the transfer rate numerically for a specified emitter as a function of its size, gate voltage, dielectric environment, and radiative lifetime [24–26]. Our approach is instead analytic and structural. We separate the total transition dipole D_X from a normalized transition form factor $F_X(q)$, and expand F_X at the small momenta $q \sim 1/z$ that the near-field filter admits. This ex-

pansion shows that, relative to the point-dipole response of the same graphene bath, the *leading* long-distance correction depends on the emitter through only a single length—the root-mean-square (rms) transition-polarization radius R_X .

We propose to extract R_X from the normalized, spacer-thickness-dependent quenching of a spectroscopically matched family of moiré-exciton lines, using the graphene gate as an internal active/Pauli-blocked reference. The aim is thus to identify *which* geometric quantity the crossover measures, and under what conditions the rich moiré structure reduces to it, rather than to compute a rate for one specific system. We then delimit this minimal picture by benchmarking it against the full RPA and by adding the anisotropic dielectric environment, the graphene plasmon, and multilayer/graphite gates.

Our goals are threefold. First, we cast the nonradiative ET rate $\Gamma_{\text{ET}}(z, \omega, \mu_F, \ell_X)$ in a single transparent expression—the overlap between the exciton near field and the electronic density response of graphene—and verify that it contains the known z^{-4} law as a limit. Second, we show that the finite size of a moiré exciton, encoded in a form factor $F_X(q)$, leaves a clear fingerprint of the transition-polarization radius R_X —related to the localization length ℓ_X once a transition-source profile is specified—in the distance dependence of the quenching. Third, we treat graphene as a gate-tunable bath, so that Pauli blocking of its electron-hole continuum allows the nonradiative channel to be electrically modulated. Throughout, we deliberately work in the long-wavelength regime. Because a distance z selects in-plane momenta $q \sim 1/z$, the essential finite-size physics is captured by a controlled low- q expansion, with the full momentum-resolved RPA used as a benchmark.

II. THEORY

A. Golden-rule rate as a near-field/loss-function overlap

The nonradiative ET rate follows from Fermi’s golden rule,

$$\Gamma_{\text{ET}} = \frac{2\pi}{\hbar} \sum_f |\langle g; f_{\text{bath}} | H_{\text{int}} | e; 0_{\text{bath}} \rangle|^2 \delta(E_f - E_0 - \hbar\omega), \quad (1)$$

where $|e\rangle, |g\rangle$ are the excited and ground states of the exciton, separated by the emission energy $\hbar\omega$, and $|0_{\text{bath}}\rangle, |f_{\text{bath}}\rangle$ are the ground and excited states of graphene. The emitter

couples to the graphene electron density ρ_G through its transition Coulomb potential ϕ_X ,

$$H_{\text{int}} = \int d^2r \rho_G(\mathbf{r}) \phi_X(\mathbf{r}, z) = \sum_{\mathbf{q}} \rho_G(-\mathbf{q}) \phi_X(\mathbf{q}, z). \quad (2)$$

The exciton is electrically neutral, so its transition charge density carries no monopole; but its dipole and higher moments are finite, and it is this oscillating transition dipole—not a net charge—that sources ϕ_X and drives the transfer. A near-field component of in-plane momentum q decays away from the emitter as e^{-qz} , so the transition probability carries a factor e^{-2qz} . Large- q components are exponentially suppressed when the emitter is far from graphene, and a given distance z probes the electronic response around $q \sim 1/z$.

The sum over graphene final states in Eq. (1) is the dynamic structure factor $S(q, \omega) = \sum_f |\langle f | \rho_G(\mathbf{q}) | 0 \rangle|^2 \delta(E_f - E_0 - \hbar\omega)$, which by the fluctuation–dissipation theorem is proportional to the imaginary part of the graphene density response function $\chi_G(q, \omega)$ [27], with $S(q, \omega) \propto [1 + n_B(\omega)] [-\text{Im} \chi_G(q, \omega)]$. At the exciton energy $\hbar\omega \gg k_B T$ the Bose factor $1 + n_B \simeq 1$ and is absorbed into the overall constant, distinct from the Pauli smearing of Eq. (11) below. The rate therefore takes the compact form

$$\Gamma_{\text{ET}}(z, \omega) \propto \int \frac{d^2q}{(2\pi)^2} |\phi_X(q, z)|^2 [-\text{Im} \chi_G(q, \omega)], \quad (3)$$

i.e. the rate is the overlap between the emitter’s near-field spectrum $|\phi_X(q, z)|^2$ and the graphene density-response loss function $L_G^{(0)}(q, \omega) \equiv -\text{Im} \chi_G(q, \omega)$, its ability to absorb energy at the same (q, ω) (the superscript (0) marks undoped graphene; gate doping enters below). In this convention ϕ_X is the *external* transition potential generated by the exciton at the graphene plane, while χ_G is the density response of graphene to that external potential. Screening internal to graphene is therefore contained in χ_G , not in ϕ_X , which avoids double counting. The corresponding screened-response kernel is made explicit in Sec. III E.

The golden rule (1) treats the emitter as spectrally narrow, which is accurate when $L_G^{(0)}$ varies slowly across the exciton linewidth. Near a sharp Pauli edge or plasmon resonance, or for a narrow low-temperature emitter, the δ -function should be replaced by a convolution of $L_G^{(0)}$ with the exciton lineshape. The same bath response also produces a dispersive exciton energy shift, the real part of the self-energy $\Sigma_X(\omega, z) \propto \int d^2q |\phi_X(q, z)|^2 \chi_G(q, \omega)$ whose imaginary part gives $\Gamma_{\text{ET}} = -2 \text{Im} \Sigma_X / \hbar$. We keep this dissipative part and neglect the Lamb-shift-like real part $\delta E_X = \text{Re} \Sigma_X$, which is largest near the Pauli edge, the plasmon, or the shortest spacings and also complicates holding the exciton energy fixed under gating.

B. Exciton form factor and the minimal model

For concreteness we first write the near-field factor for a perpendicular point transition dipole, which produces the potential $\phi_X(q, z) \propto e^{-qz}$ —the dipole factor q cancels the $1/q$ of the 2D Coulomb kernel—so $|\phi_X|^2 \propto e^{-2qz}$. An in-plane transition dipole gives the same exponential e^{-2qz} after angular averaging and differs only by an orientation-dependent prefactor (Appendix A). The distance and localization dependence discussed below are therefore insensitive to the precise optical-dipole orientation. (Interlayer moiré excitons additionally carry a *static* out-of-plane electric dipole. It arises from the electron–hole layer separation, which also reduces the optical overlap and lengthens the radiative lifetime [28], and produces a linear Stark shift [29] that should be held fixed or compensated when the graphene Fermi level is tuned. This static dipole is distinct from the optical transition dipole considered here.)

A moiré exciton is instead spatially extended, and its coupling to graphene is set by its transition-polarization density $\mathbf{P}_X(\mathbf{R})$, a function of the in-plane center-of-mass coordinate $\mathbf{R}$ measured from the local moiré-potential minimum—the source of the near field (the transition charge density is $\rho_X = -\nabla \cdot \mathbf{P}_X$). Write $\mathbf{P}_X = \mathbf{u} P_X$, with $\mathbf{u}$ the unit vector along the transition dipole. For an optically bright state [$P_X(\mathbf{0}) \neq 0$] we *define* the total transition dipole $D_X \equiv P_X(\mathbf{0})$ and the normalized form factor $F_X(q) \equiv P_X(\mathbf{q})/P_X(\mathbf{0})$, so that $P_X(\mathbf{q}) = D_X F_X(q)$ with $F_X(0) = 1$ identically. This split is exact and separates the oscillator strength D_X from a form factor that encodes the momentum filtering; the localization length ℓ_X enters only through the single-envelope model below, where $F_X(q; \ell_X)$ is set by the center-of-mass envelope and $D_X(\ell_X)$ acquires its coherent-area scaling. The optical transition polarization is moreover *linear* in the center-of-mass amplitude—within the envelope-function approximation, $\mathbf{P}_X(\mathbf{R}) = \langle 0 | \hat{\mathbf{P}}(\mathbf{R}) | X \rangle \propto \psi_X(\mathbf{R})$ [30] (Appendix B), with $\hat{\mathbf{P}}$ the polarization-density operator—so the source profile follows ψ_X , not the probability density $|\psi_X|^2$. For a normalized Gaussian envelope $\psi_X(\mathbf{R}) \propto e^{-R^2/2\ell_X^2}$ this gives (Appendix B)

$$|F_X(q; \ell_X)|^2 = e^{-q^2 \ell_X^2}, \quad |D_X(\ell_X)|^2 \propto \ell_X^2, \quad (4)$$

where the ℓ_X^2 is the coherent (giant-oscillator-strength) enhancement that arises from summing the transition dipole coherently over the localization area [31–35].

The microscopic optical factors—the relative-coordinate wave function at zero electron–

hole separation, interlayer overlap, and valley/spin selection rules—set the local dipole prefactor, whereas the center-of-mass envelope fixes both the coherent-area enhancement of D_X and the momentum filtering through F_X , suppressing components with $q \gtrsim 1/\ell_X$.

Two approximations underlie the identification of F_X with the center-of-mass-envelope form factor. First, we place the transition polarization in a single effective 2D plane. For an interlayer exciton with appreciable layer-resolved transition amplitudes the factor $e^{-qz} P_X(q)$ becomes a coherent sum $\sum_\alpha e^{-qz_\alpha} P_{X,\alpha}(q)$ over the constituent layers α , where z_α is the graphene-to-layer distance and $P_{X,\alpha}$ the transition-polarization amplitude in layer α . With the layers separated by $\delta z \sim 0.6\text{--}0.7$ nm, this vertical spread is non-negligible ($e^{-q\delta z} \sim 0.8$ at $q \sim 0.4$ nm⁻¹) once $q \sim 1/z$ grows.

Second, we treat the total transition dipole D_X as q -independent, which holds while the microscopic optical factors vary weakly over the near-field window $q \sim 1/z$. The relevant internal scale here is the electron–hole relative size $a_X \sim 1\text{--}2$ nm [12]—the internal exciton radius, distinct from the center-of-mass envelope ℓ_X . This approximation can fail at the smallest spacings. When $q \sim 1/z$ approaches $1/a_X$, the transition dipole acquires its own microscopic form factor, $D_X \rightarrow D_X M_{\text{micro}}(q)$ with $M_{\text{micro}}(0) = 1$, set by the internal (relative-coordinate) exciton wave function—the internal analogue of the envelope factor F_X . Both M_{micro} and the vertical (interlayer) factor above also suppress high q and can therefore mimic an additional finite lateral size in the graphene response. Graphene therefore responds to the *combined* transition-source form factor, so isolating the lateral envelope radius R_X requires the vertical and microscopic factors to be known or negligible.

For this extended source the near-field potential is $\phi_X(q, z) \propto D_X F_X(q; \ell_X) e^{-qz}$ —the point-dipole factor e^{-qz} times the source form factor $D_X F_X$ —so that $|\phi_X(q, z)|^2 \propto |D_X|^2 |F_X(q; \ell_X)|^2 e^{-2qz}$. The neutral-graphene interband response at small q is $-\text{Im } \chi_G \propto q^2/\omega$ (Appendix A). Inserting both into Eq. (3) with the planar measure $d^2q = 2\pi q dq$ reduces it to a single radial integral,

$$\Gamma_{\text{ET}} = C_G(\omega) |D_X(\ell_X)|^2 \int_0^{q_c} dq q^3 e^{-2qz} |F_X(q; \ell_X)|^2. \quad (5)$$

Here $C_G(\omega)$ collects the graphene-response and dielectric-environment prefactors—not the emitter oscillator strength, which is contained in $|D_X|^2$. It is independent of z and ℓ_X and therefore sets only the overall scale.

The upper limit $q_c \simeq \omega/v_F$ is the kinematic cutoff above which a Dirac electron–hole

pair can no longer conserve energy and momentum. With the graphene Dirac velocity $\hbar v_F = 0.658 \text{ eV nm}$ [36] and $\hbar\omega = 1.5 \text{ eV}$ one has $q_c \approx 2.3 \text{ nm}^{-1}$ ($1/q_c \approx 0.4 \text{ nm}$). This sharp cutoff is only a schematic stand-in for the kinematic edge. The exact response [Eq. (A1)] has an integrable square-root singularity at $q = q_c$, but the near-field factor e^{-2qz} suppresses that edge exponentially, so for the experimentally relevant spacings $z \gtrsim 2 \text{ nm}$ —several times $1/q_c$, so that $e^{-2q_c z} \lesssim 10^{-4}$ at the cutoff—neither the cutoff nor the precise edge shape affects the result.

A final filter is environmental rather than intrinsic to the exciton. Besides the exciton's own size, the surrounding dielectric—the TMDC layers, the hBN spacer, and their interfaces—also shapes the near field on its way to graphene. We collect this into an *electrostatic transfer function* $T(q, \omega)$, the factor by which the stack transmits each momentum component of the potential to graphene, so that $\phi_X \propto T(q, \omega) e^{-qz} P_X(q)$ and the rate reads $\Gamma_{\text{ET}} \propto \int q dq e^{-2qz} |T(q, \omega)|^2 |F_X(q)|^2 L_G^{(0)}(q, \omega)$. Microscopically, $T(q, \omega)$ carries the TMDC background polarizability [37], Keldysh-type 2D screening [38, 39], and interface and finite-slab reflections. Because the measurement samples only $q \sim 1/z$, what matters is how $T(q, \omega)$ varies across that window. A flat $T(q, \omega)$ merely rescales the rate. A $T(q, \omega)$ that falls with q over $q \sim 1/z$ instead suppresses the high- q near field exactly as the exciton form factor $F_X(q)$ does, so this environmental q -dependence can be misread as extra exciton size and bias the extracted R_X . A Keldysh form $T(q, \omega) \sim (1 + r_* q)^{-1}$, for instance, adds an r_*/z term of even lower order than the R_X^2/z^2 size correction. Any known $T(q, \omega)$ must therefore be folded into the point-source kernel, as $|T(q, \omega)|^2 |F_X(q)|^2$, before R_X is extracted. The minimal kernel (5) takes $T(q, \omega)$ smooth over $q \sim 1/z$ and absorbs its $q \rightarrow 0$ value into C_G .

For the extended coherent exciton the distance dependence is governed by $|F_X(q; \ell_X)|^2 = e^{-q^2 \ell_X^2}$, while $|D_X|^2 \propto \ell_X^2$ multiplies the whole rate. Two consequences should be kept separate. For a *fixed* exciton D_X is a common multiplicative factor, so the normalized distance dependence and gate modulation are independent of the oscillator-strength model, while the finite-size deviation from the point-source curve is controlled by $F_X(q)$. Comparing *different* localization lengths additionally involves $D_X(\ell_X)$. As a transparent benchmark that isolates momentum filtering at fixed oscillator strength, we also use a fixed-total-dipole model in which D_X is held constant and the source is a normalized Gaussian transition-polarization profile, so that $|F_X(q; \ell_X)|^2 = e^{-q^2 \ell_X^2/2}$. For this Gaussian parametrization the profile width coincides numerically with that of $|\psi_X|^2$, but it is not interpreted as a

probability-density source. This benchmark recovers the point-dipole z^{-4} law for every ℓ_X at large z .

At fixed total transition dipole D_X , the form-factor kernel approaches the point-dipole result as $\ell_X \rightarrow 0$. The point-dipole molecule of Swathi and Sebastian ($F_X \rightarrow 1$) recovers, for $q_c \rightarrow \infty$,

$$\Gamma_{\text{ET}} \propto \int_0^\infty dq q^3 e^{-2qz} = \frac{3}{8z^4}, \quad (6)$$

the z^{-4} law of Refs. [1, 2]. In the coherent-envelope model, by contrast, $|D_X|^2 \propto \ell_X^2$, so the mathematical limit $\ell_X \rightarrow 0$ sends $\Gamma_{\text{ET}} \rightarrow 0$ and does not describe a point molecule. It lies outside the weak-confinement envelope description. For $\ell_X > 0$ the Gaussian factor removes the high- q part of the near field, so that the rate falls below its own z^{-4} asymptote once $z \lesssim \ell_X$.

C. General form factor: low- q expansion and the moiré reduction

The Gaussian envelope above is a convenient special case. The near-field filter makes the finite-size physics far more general. For an arbitrary scalar transition-polarization amplitude $p_X(\mathbf{R})$ —the source magnitude introduced above—with $D_X = \int d^2R p_X$ and $F_X(q) = D_X^{-1} \int d^2R p_X(\mathbf{R}) e^{-i\mathbf{q}\cdot\mathbf{R}}$ (Appendix B), centering the source ($\int d^2R \mathbf{R} p_X = 0$) and Taylor-expanding the plane-wave factor $e^{-i\mathbf{q}\cdot\mathbf{R}}$ in small q gives, for an isotropic bright state,

$$F_X(q) = 1 - \frac{1}{4}q^2 \langle R^2 \rangle_P + O(q^4), \quad (7)$$

$$\langle R^2 \rangle_P \equiv \frac{\int d^2R R^2 p_X(\mathbf{R})}{\int d^2R p_X(\mathbf{R})},$$

where $\langle R^2 \rangle_P$ is the second radial moment of the source. For a nodeless, phase-uniform s-like bright state we fix the global phase so that $D_X > 0$ and $p_X(\mathbf{R})$ is real. The second moment $\langle R^2 \rangle_P$ then defines a positive transition-polarization radius $R_X \equiv \langle R^2 \rangle_P^{1/2}$. This is the second-moment radius of the optical transition polarization. It need not equal the rms radius of the center-of-mass probability density $|\psi_X|^2$, and converting between the two requires a source model.

It follows that $|F_X(q)|^2 = 1 - \frac{1}{2}q^2 \langle R^2 \rangle_P + O(q^4)$, and inserting this into the rate gives the leading long-distance correction to the point-dipole law,

$$\Gamma_{\text{ET}}(z) = \frac{3C_G |D_X|^2}{8z^4} \left[1 - \frac{5}{2} \frac{\langle R^2 \rangle_P}{z^2} + O\left(\frac{\langle R^4 \rangle_P}{z^4}\right) \right], \quad (8)$$

valid for $z \gg R_X, q_c^{-1}$, where the next-order term involves the *independent* fourth moment $\langle R^4 \rangle_P$. Within the minimal q^2 bath kernel, this *first* emitter-induced deviation from the z^{-4} law is governed by R_X^2 alone, independently of the detailed source, with higher moments entering only in the full crossover at $z \sim R_X$. More generally, this universality applies to the ratio to the point-dipole response evaluated with the same bath. For the Gaussian envelope $\langle R^2 \rangle_P = 2\ell_X^2$ (coherent) or ℓ_X^2 (fixed-dipole benchmark), consistent with Eq. (4), so in R_X the two Gaussian form factors coincide and their crossover curves collapse, with a half-suppression at $z \simeq 1.6 R_X$ (Sec. III B). For a general non-Gaussian source this collapse and half-suppression are not universal—only the leading deviation is. Equation (8) therefore gives a model-independent R_X only for $z \gg R_X$; the stronger crossover requires the transition-polarization profile or higher moments.

This two-term expansion converges slowly. For a Gaussian source the exact ratio $\Gamma_{\text{ET}}/\Gamma_{\text{ET}}^{\text{pd}}$ of the finite-size rate to the point-dipole rate $\Gamma_{\text{ET}}^{\text{pd}} (F_X \rightarrow 1 \text{ at the same } D_X)$ is 0.62, 0.82, and 0.91 at $z/R_X = 2, 3.3$, and 5, whereas Eq. (8) gives 0.38, 0.78, and 0.90. The leading correction is thus quantitative only for $z/R_X \gtrsim 4$ –5. In the crossover $z \sim R_X$, where the signal is largest, the full form factor or a controlled multi-moment expansion must be used. The next term is $[\frac{105}{32}\langle R^2 \rangle_P^2 + \frac{105}{64}\langle R^4 \rangle_P]/z^4$, reducing for a Gaussian to $+\frac{105}{16}R_X^4/z^4$.

This universality also presumes a fixed bath response. The bare graphene kernel is itself weakly q -dependent. Expanding Eq. (A1) gives $-\text{Im } \chi_0 \propto (q^2/\omega)[1 + \frac{1}{2}(v_F q/\omega)^2 + \dots]$, which adds a bath term $+\frac{5}{2}\lambda_\omega^2/z^2$ inside the bracket of Eq. (8), with $\lambda_\omega \equiv \hbar v_F/\hbar\omega \simeq 0.44$ nm at $\hbar\omega = 1.5$ eV. The raw deviation from a pure z^{-4} power law therefore carries $\frac{5}{2}(\lambda_\omega^2 - R_X^2)/z^2$, whereas the ratio to the point-dipole rate evaluated with the same bath, $\Gamma_{\text{ET}}(z)/\Gamma_{\text{ET}}^{\text{pd}}(z) = 1 - \frac{5}{2}R_X^2/z^2 + \dots$ (with $z \rightarrow z_{\text{eff}} = \gamma z$ once the spacer-anisotropy factor γ of Sec. III F is included), cancels the bath term and isolates R_X . The bath correction is $\sim 2\%$ of the size term for $R_X = 3$ nm but grows to ~ 10 –20% for $R_X \lesssim 1$ –1.5 nm, so R_X is best extracted from a fit that retains the known graphene kernel—equivalently the ratio to the point-dipole rate—rather than from the deviation from a bare z^{-4} law.

The single-radius reduction presumes an isolated optically bright state [$D_X = P_X(0) \neq 0$] and does not apply to dark, nodal, or phase-textured states, whose coupling is instead set by the leading nonvanishing multipole. Because graphene accepts finite in-plane momentum, it can moreover couple to momentum-dark excitons [$P_X(0) = 0, P_X(\mathbf{Q}) \neq 0$] that are invisible in far-field PL. Their graphene-induced depletion can affect a bright line even at fixed R_X ,

but requires an unnormalized finite- q , multilevel treatment deferred here.

Moiré excitons carry microscopic structure that this expansion makes explicit. Physically, the transition polarization is the slow center-of-mass envelope $\psi_X(\mathbf{R})$ times a *local* transition dipole $\mathbf{d}(\mathbf{R})$ —a vector field whose strength, direction, and valley/spin character follow the atomic registry and therefore repeat with the moiré period. Its Fourier series over the moiré reciprocal lattice, $\mathbf{d}(\mathbf{R}) = \sum_{\mathbf{G}_M} \mathbf{d}_{\mathbf{G}_M} e^{i\mathbf{G}_M \cdot \mathbf{R}}$, gives

$$\begin{aligned} \mathbf{P}_X(\mathbf{R}) &= \psi_X(\mathbf{R}) \sum_{\mathbf{G}_M} \mathbf{d}_{\mathbf{G}_M} e^{i\mathbf{G}_M \cdot \mathbf{R}}, \\ \mathbf{P}_X(\mathbf{q}) &= \sum_{\mathbf{G}_M} \mathbf{d}_{\mathbf{G}_M} \tilde{\psi}_X(\mathbf{q} - \mathbf{G}_M), \end{aligned} \tag{9}$$

an envelope-times-Bloch (wave-packet) form in which the coefficients $\mathbf{d}_{\mathbf{G}_M}$ encode the registry-dependent optical selection rules, valley and spin structure, and interlayer hybridization at each moiré reciprocal vector $\mathbf{G}_M$ [19, 20], while $\tilde{\psi}_X$ is the center-of-mass envelope. The cell average $\mathbf{d}_0$ of $\mathbf{d}(\mathbf{R})$ —the coarse-grained, bright dipole—supplies the microscopic bright-dipole factor; in this reduction the total transition dipole is $D_X = |\mathbf{d}_0| \tilde{\psi}_X(\mathbf{0})$, including the coherent-envelope contribution $\tilde{\psi}_X(\mathbf{0}) \propto \ell_X$ that yields the giant oscillator strength $|D_X|^2 \propto \ell_X^2$. The higher harmonics $\mathbf{d}_{\mathbf{G}_M \neq 0}$ describe how $\mathbf{d}(\mathbf{R})$ varies within a moiré cell.

For a bright, s -like ground state, two conditions make the $\mathbf{G}_M \neq \mathbf{0}$ (Umklapp) terms negligible at the sampled momenta $q \sim 1/z$. First, $z|\mathbf{G}_M| \gg 1$: the near-field filter e^{-qz} passes only $q \sim 1/z \ll |\mathbf{G}_M|$, so components displaced by a nonzero $\mathbf{G}_M$ are exponentially suppressed. Second, $\ell_X |\mathbf{G}_M| \gg 1$: an envelope spread over many moiré cells ($\ell_X \gg 1/|\mathbf{G}_M|$) is correspondingly narrow in momentum (width $\sim 1/\ell_X \ll |\mathbf{G}_M|$), so the copies $\tilde{\psi}_X(\mathbf{q} - \mathbf{G}_M)$ centered at each $\mathbf{G}_M$ do not overlap, and those with $\mathbf{G}_M \neq \mathbf{0}$ leave negligible weight near $q \sim 0$. When both hold, only the $\mathbf{G}_M = \mathbf{0}$ term survives and $\mathbf{P}_X(\mathbf{q}) \simeq \mathbf{d}_{\mathbf{G}=0} \tilde{\psi}_X(\mathbf{q}) \equiv D_X F_X(q)$. The microscopic moiré physics enters through the prefactor D_X , whereas the low- q distance dependence is governed by the envelope radius R_X alone.

The reduction further requires the bright $\mathbf{G}_M = \mathbf{0}$ coefficient to dominate. It holds only when $|\mathbf{d}_{\mathbf{G}_M} \tilde{\psi}_X(\mathbf{q} - \mathbf{G}_M)| \ll |\mathbf{d}_0 \tilde{\psi}_X(\mathbf{q})|$ across the near-field window $q \sim 1/z$, which can fail for registries with a small $\mathbf{d}_0$ even when the envelope factor is exponentially suppressed. This is the precise sense in which a moiré exciton reduces to a single length scale, but the conditions are not automatic. They hold for typical short-period moirés with sufficiently smooth ground-state envelopes. A moiré period $a_M = 10$ nm gives $|\mathbf{G}_M| = 4\pi/(\sqrt{3}a_M) \simeq$

0.73 nm^{-1} , so the Umklapp weight $e^{-2|\mathbf{G}_M|z}$ is $\sim 7 \times 10^{-4}$ at $z = 5 \text{ nm}$ and $\sim 10^{-2}$ at $z = 3 \text{ nm}$, while $\ell_X|\mathbf{G}_M| \simeq 7$ for $\ell_X = 10 \text{ nm}$ but only $\simeq 2$ for $\ell_X = 3 \text{ nm}$. The reduction can fail, however, for long-period moirés or very thin spacers ($z \lesssim 2 \text{ nm}$), and for tightly localized, excited, phase-textured, momentum-dark, or strongly registry-modulated states—cases in which the $\mathbf{G}_M \neq 0$ terms, higher envelope moments, or the full transition density must be retained rather than a single R_X .

The scalar reduction further assumes a single bright polarization channel with a common dipole direction $\mathbf{u}$. Multiple or spatially varying channels replace F_X and $\langle R^2 \rangle_P$ by the corresponding tensors. Equation (9) is itself a single-envelope ansatz. A microscopic continuum model can entangle the moiré harmonics, layer composition, and registry-dependent optical dipole into a superposition over several harmonics whose transition density does not factor into a smooth envelope times a $\mathbf{G}_M = 0$ dipole, in which case the full $P_X(q)$ must be retained. Such a state-resolved microscopic evaluation is beyond the universal long-wavelength theory developed here.

In summary, the reduction of the graphene coupling to a single transition-polarization radius R_X rests on several conditions: an isolated, phase-uniform bright state; a transition polarization effectively confined to one plane; a microscopic and environmental transfer function that varies slowly over $q \sim 1/z$; negligible gate- or spacer-induced modification of the exciton state; and, for the extraction protocol below, approximately single-exponential population decay. When these hold, proximity quenching measures the low- q transition-polarization form factor, and its leading finite-size correction is governed by R_X alone. When they fail, the full layer-, momentum-, and state-resolved transition density must be retained, and the single- R_X reduction becomes a hypothesis to be tested against a microscopic model.

D. Gate tunability: Pauli blocking

Doping graphene to a Fermi level μ_F blocks interband transitions whose final state is already occupied. A vertical interband Dirac transition of energy $\hbar\omega$ requires an empty final state, available only when $\hbar\omega > 2|\mu_F|$. This gate-tunable Pauli blocking of the interband absorption is well documented in graphene optical spectroscopy [40, 41]. As a first step we adopt the sharp Pauli-blocking model

$$L_G(q, \omega, \mu_F) = L_G^{(0)}(q, \omega) \Theta(\hbar\omega - 2|\mu_F|), \quad (10)$$

where $L_G^{(0)}(q, \omega)$ is the neutral-graphene loss function of Eq. (3) and $L_G(q, \omega, \mu_F)$ its Pauli-blocked form (the extra argument μ_F marks the doped response).

The exact interband threshold tilts with momentum, $\hbar\omega > 2|\mu_F| - \hbar v_F q$. The near-field filter selects $q \sim 1/z$, which is smaller than, but not always negligible compared with, the Fermi momentum k_F . In the optical-limit approximation the blocking factor is taken to be q -independent and multiplies Γ_{ET} as a whole, suppressing the interband contribution above $|\mu_F| = \hbar\omega/2$. The momentum tilt is not entirely negligible at the shortest spacings, however. Blocking the representative near-field momentum $q_{\text{peak}} = 3/2z$ (where the rate weight $q^3 e^{-2qz}$ peaks) requires $|\mu_F| \gtrsim (\hbar\omega + \hbar v_F q_{\text{peak}})/2$, i.e. ≈ 0.85 and 0.91 eV at $z = 5$ and 3 nm (versus 0.75 eV at $q = 0$), so the sharp $q = 0$ threshold underestimates the gate voltage needed for complete blocking at short range, as the full momentum-resolved RPA polarizability of Sec. III E confirms.

At finite temperature the interband edge is thermally smeared. The $q \rightarrow 0$ (optical) graphene interband response gives the rigorous form [42]

$$\Theta(\hbar\omega - 2|\mu_F|) \longrightarrow \frac{1}{2} \sum_{\pm} \tanh \frac{\hbar\omega \pm 2|\mu_F|}{4k_B T}, \quad (11)$$

which reduces to the step as $T \rightarrow 0$ and broadens the threshold over $\sim 4k_B T$ (≈ 0.1 eV at room temperature). A more complete treatment would use the full Dirac-electron RPA response [27, 43], including the intraband (Drude/plasmon) channel. We develop this in Sec. III E.

E. From rates to PL observables

The total exciton decay rate is $\Gamma_{\text{tot}} = \Gamma_{\text{rad}} + \Gamma_{\text{nr}}^0 + \Gamma_{\text{ET}}$, with Γ_{rad} the radiative rate and Γ_{nr}^0 the intrinsic (graphene-free) nonradiative rate. The excited-state population decays at the total rate Γ_{tot} —every channel, radiative or not, removes an exciton—so the PL lifetime is $\tau_{\text{PL}} = 1/\Gamma_{\text{tot}}$, whereas only a fraction $\Gamma_{\text{rad}}/\Gamma_{\text{tot}}$ of those decays emit a photon; the PL intensity is therefore proportional to this quantum yield. Here I_{PL} is the spectrally and temporally integrated PL intensity under fixed excitation and collection conditions. In a pulsed (time-resolved) measurement the PL decays as $I_{\text{PL}}(t) \propto \Gamma_{\text{rad}} N_0 e^{-\Gamma_{\text{tot}} t}$, with N_0 the pulse-created initial population: the graphene channel raises Γ_{tot} and thus shortens the lifetime τ_{PL} , but leaves the $t = 0$ peak height $\Gamma_{\text{rad}} N_0$ unchanged at fixed N_0 . In a decay

trace, graphene quenching then appears as a faster decay and a smaller area under the curve, but not as a lower initial height at $t = 0$.

The absolute transfer rate carries an overall microscopic prefactor that we prefer not to fix. We therefore measure spacings in units of the *graphene quenching distance* d_0 , the graphene analogue of the Förster radius: the spacing at which the *point-dipole* transfer rate $\Gamma_{\text{ET}}^{\text{pd}}$ —evaluated with the same total transition dipole D_X ($F_X \rightarrow 1$)—equals the emitter’s intrinsic (graphene-free) decay rate, $\Gamma_{\text{ET}}^{\text{pd}}(d_0) = \Gamma_0(\ell_X) \equiv \Gamma_{\text{rad}}(\ell_X) + \Gamma_{\text{nr}}^0(\ell_X)$. At this spacing a point emitter’s total rate is doubled, $\Gamma_{\text{tot}} = \Gamma_0 + \Gamma_{\text{ET}}^{\text{pd}} = 2\Gamma_0$, so its lifetime ($\tau_{\text{PL}} = 1/\Gamma_{\text{tot}}$) and integrated PL ($\propto 1/\Gamma_{\text{tot}}$) fall to half their graphene-free values. Referencing distances to d_0 absorbs the overall constant entirely, so finite-size effects enter only as corrections relative to it.

The value $d_0 \sim 15$ nm used in the figures is illustrative: it sets the overall scale and is of the same order as the graphene energy-transfer distances measured for quantum emitters [4, 7]. All trends below rescale for a different, experimentally determined d_0 , so the predictions are statements about scaling and device design rather than about an absolute rate. Writing $x \equiv \Gamma_{\text{ET}}/\Gamma_0$, both observables, measured relative to the graphene-free emitter, reduce to the same function

$$\frac{\tau_{\text{PL}}}{\tau_0} = \frac{I_{\text{PL}}}{I_0} = \frac{1}{1+x}, \quad x = \frac{\Gamma_{\text{ET}}(z, \ell_X, \mu_F)}{\Gamma_0(\ell_X)}. \quad (12)$$

Here $\tau_0 = 1/\Gamma_0$ and I_0 are the lifetime and integrated PL intensity of the same emitter without graphene.

The relative PL intensity and lifetime should therefore coincide whenever only Γ_{ET} varies—a directly testable prediction. Equation (12) assumes the radiative rate Γ_{rad} is unchanged by graphene. Graphene does modify Γ_{rad} by reshaping the emitter’s photonic local density of states; capturing that shift would require the full electromagnetic dyadic Green’s function of the layered stack—the object that fixes a dipole’s decay rate near any structure. In the strong-quenching regime considered here we assume that this near-field nonradiative channel Γ_{ET} dominates over the graphene-induced radiative-LDOS correction, which we therefore omit. Equation (12) is applied with a common d_0 for variations of z or μ_F at *fixed* ℓ_X , for which D_X , Γ_{rad} , and Γ_{nr}^0 are unchanged. When different localization lengths are compared, the reference $\Gamma_0(\ell_X)$ and $d_0(\ell_X)$ —and hence the oscillator strength—must be calibrated separately for each exciton state.

III. RESULTS

All curves below use $\hbar\omega = 1.5$ eV (a representative TMDC interlayer-exciton energy), $\hbar v_F = 0.658$ eV nm, and, where a scale is needed, a quenching distance $d_0 = 15$ nm and room temperature $T = 300$ K for the interband edge [Eq. (11)].

A. Benchmark: recovery of the z^{-4} law

Figure 2 shows the point-dipole rate $\Gamma_{\text{ET}}^{\text{pd}}(z) = C_G(\omega) |D_X|^2 \int_0^{q_c} dq q^3 e^{-2qz}$. On a log-log scale it follows a slope of -4 over the nanometric, nonretarded range relevant here. A power-law fit for $z > 20$ nm gives an exponent of -4.0000 , confirming that the numerical evaluation recovers the analytic limit Eq. (6) of Refs. [1, 2, 4]. The finite cutoff q_c bends the curve below the z^{-4} asymptote only for $z \lesssim 1/q_c \sim 0.4$ nm, far below any realistic hBN spacer, so the cutoff plays no role in practice.

B. Finite-size effect: distance dependence as a probe of localization

Figure 3 contrasts the two source models. The coherent exciton is the physical case; the fixed-dipole benchmark is a deliberately different foil, included to separate the model-independent content (the extraction of R_X) from the fingerprint of the coherent giant oscillator strength (the absolute ℓ_X -dependence of the rate). In both, $\Gamma_{\text{ET}}(z)$ tracks the point-dipole z^{-4} law at large spacing ($z \gg \ell_X$), where the emitter looks point-like. Once the spacing drops below the exciton size ($z \lesssim \ell_X$), the finite source suppresses the short-wavelength ($q \sim 1/z$) near field and the rate flattens, deviating from z^{-4} . This distance-sweep crossover is the robust signature of the transition-polarization radius. Because it comes from the shape $|F_X(q)|^2$ and not from D_X , it is independent of the oscillator-strength model.

The half-suppression point—where the rate falls to half of the point-dipole rate evaluated with the same bath—sits at $z \approx 1.6 \ell_X$ for the fixed-dipole benchmark and $z \approx 2.25 \ell_X$ for the coherent exciton. This apparent model dependence is an artifact of the nominal ℓ_X . The two Gaussian models assign different transition-polarization radii to the same ℓ_X [$\langle R^2 \rangle_P = \ell_X^2$ and $2\ell_X^2$, Eq. (7)], and because in the physical radius $R_X = \langle R^2 \rangle_P^{1/2}$ their form factors coincide, both collapse onto the same curve with a half-suppression at $z \approx 1.6 R_X$ [panel (d)]. What is universal for an *arbitrary* source, however, is only the leading long-

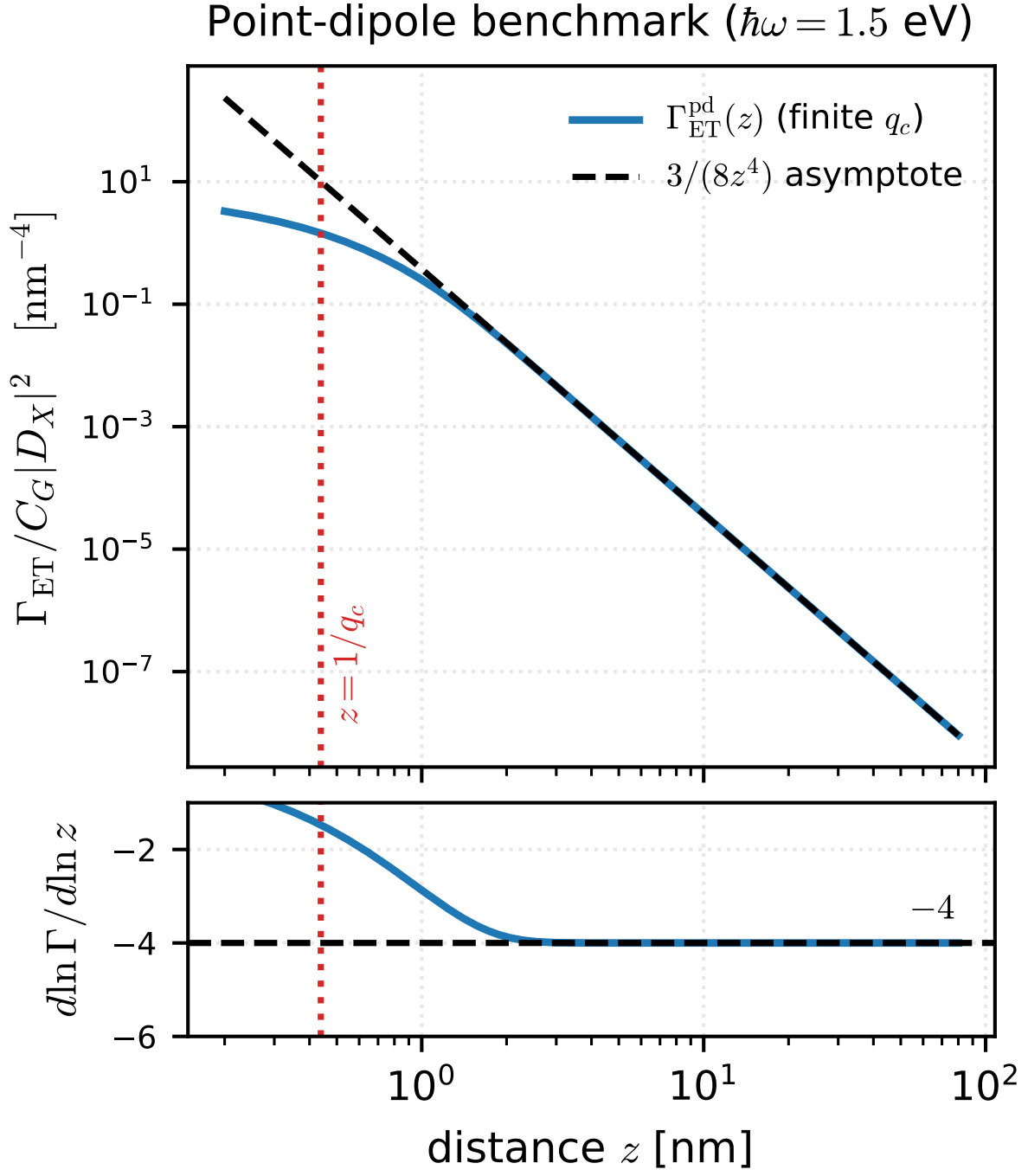


FIG. 2. (Color online) Benchmark for a point dipole ($F_X = 1$, fixed D_X). Top: ET rate (solid) and the $3/(8z^4)$ asymptote (dashed); the vertical line marks $z = 1/q_c$. Bottom: local logarithmic slope, approaching -4 at large z .

distance deviation $\propto R_X^2$ [Eq. (8)], while the full crossover can involve higher moments. The clean, model-independent observable is therefore R_X , extracted from the leading large- z deviation from the point-dipole response evaluated with the same bath, rather than the convention-dependent envelope length ℓ_X .

In the *absolute* rate, by contrast, the two models differ in their dependence on ℓ_X . In the fixed-dipole benchmark [panel (b)] the curves merge onto a common z^{-4} asymptote and Γ_{ET} decreases monotonically with ℓ_X . In the coherent model [panel (a)] the giant oscillator strength $|D_X|^2 \propto \ell_X^2$ makes the long-range amplitude grow with ℓ_X , so the curves separate by ℓ_X^2 and cross. Small excitons quench more strongly at short range, large excitons at long range. At fixed distance this produces a *non-monotonic* size dependence [panel (c)], with $\Gamma_{\text{ET}}(\ell_X)$ peaking near $\ell_X \approx z$, a distinctive prediction of the coherent picture. This model dependence of the absolute rate shapes how one should measure the exciton size. Comparing excitons of *different* ℓ_X requires the oscillator-strength law $D_X(\ell_X)$, which differs between the fixed-dipole and fully-coherent limits and so cannot cleanly isolate the size. Sweeping the spacing z for a *fixed* transition source, by contrast, leaves $D_X(\ell_X)$ out entirely: the crossover position measures R_X directly. And because a moiré exciton's ℓ_X is intrinsic and twist-tunable—unlike the fixed size of a quantum dot [24]—this distance sweep becomes a tunable probe of R_X (and, once a source model is chosen, of ℓ_X).

C. Gate dependence: Pauli-blocked bath

Figure 4(a) shows the suppression of Γ_{ET} with graphene Fermi level. In the minimal model the interband channel is strongly suppressed as $2|\mu_F|$ approaches $\hbar\omega$, i.e. $|\mu_F| \rightarrow \hbar\omega/2$ (0.65, 0.75, and 0.85 eV for $\hbar\omega = 1.3, 1.5, 1.7$ eV), thermally broadened over $\sim 4k_B T$ at finite temperature [Eq. (11)]. Residual intraband absorption, disorder, and many-body effects make the suppression strong but not complete in practice. The $(\hbar\omega, |\mu_F|)$ map [panel (b)] separates an active region, where graphene is a strong quencher, from a Pauli-blocked region where its interband absorption is strongly suppressed. This is the mechanism behind electrically controlled relaxation pathways [7], here applied to moiré excitons.

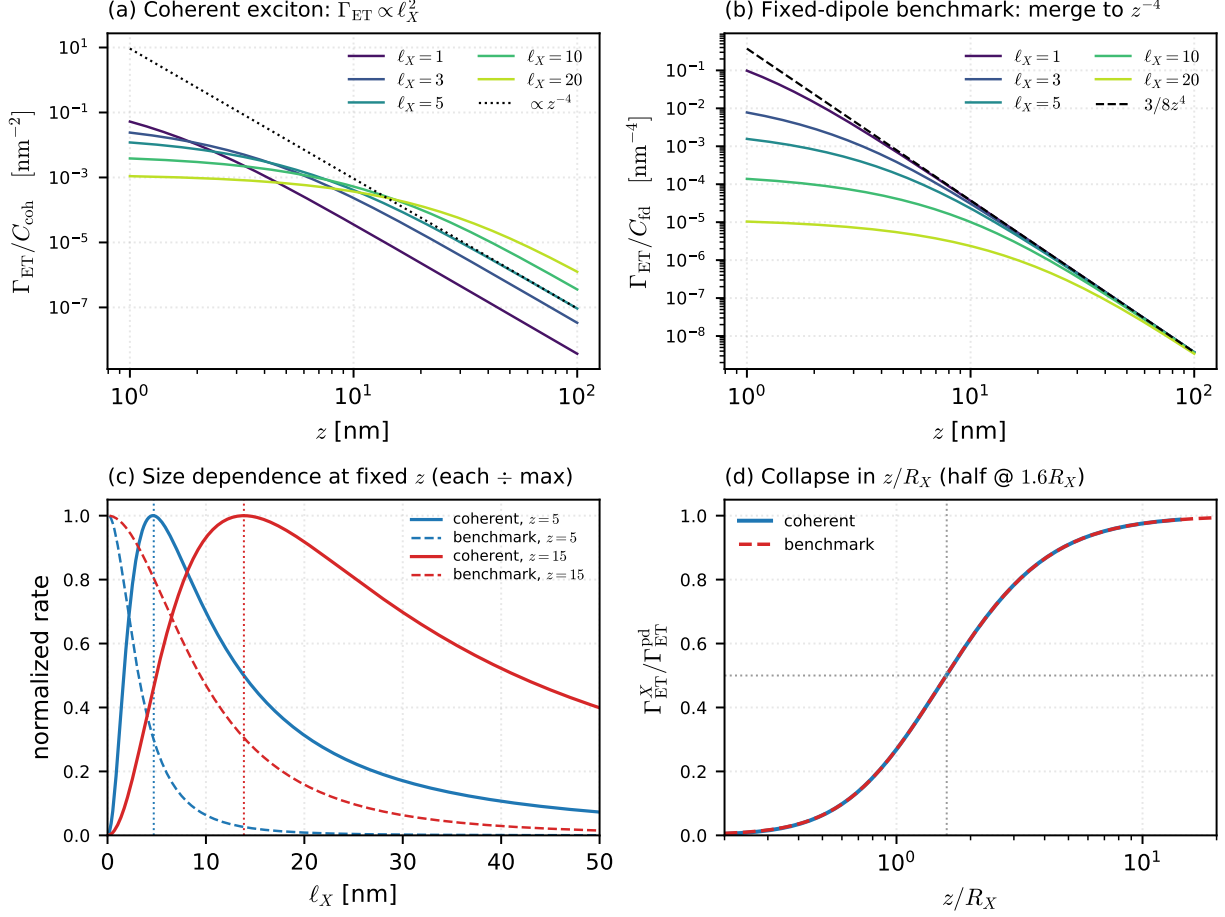


FIG. 3. (Color online) Finite-size effect: coherent exciton versus fixed-dipole benchmark. (a) Coherent model ($|D_X|^2 \propto \ell_X^2$, $|F_X|^2 = e^{-q^2 \ell_X^2}$): the rate (in units of the constant coherent prefactor C_{coh}) splits by ℓ_X^2 and the curves cross—small excitons dominate at short range, large excitons at long range—while every curve keeps the z^{-4} slope (dotted). (b) Fixed-dipole benchmark (D_X const, prefactor C_{fd}): all curves merge onto the point-dipole $3/(8z^4)$ asymptote (dashed). (c) Size dependence at fixed z (each curve normalized to its own maximum): the coherent rate is *non-monotonic*, peaking near $\ell_X \approx z$ (dotted verticals), whereas the benchmark decreases monotonically. (d) Momentum-filtering shape (rate divided by the point-dipole rate evaluated with the same bath, $\Gamma_{\text{ET}}^X/\Gamma_{\text{ET}}^{\text{pd}}$) versus z/R_X , with $R_X = \langle R^2 \rangle_P^{1/2}$ the rms transition-polarization radius ($R_X = \ell_X$ and $\sqrt{2}\ell_X$ for the two models): expressed in R_X the two Gaussian curves collapse, with a half-suppression at $z \simeq 1.6 R_X$; this exact collapse is specific to the two Gaussian models, whereas for a general source only the leading long-distance coefficient is universal.

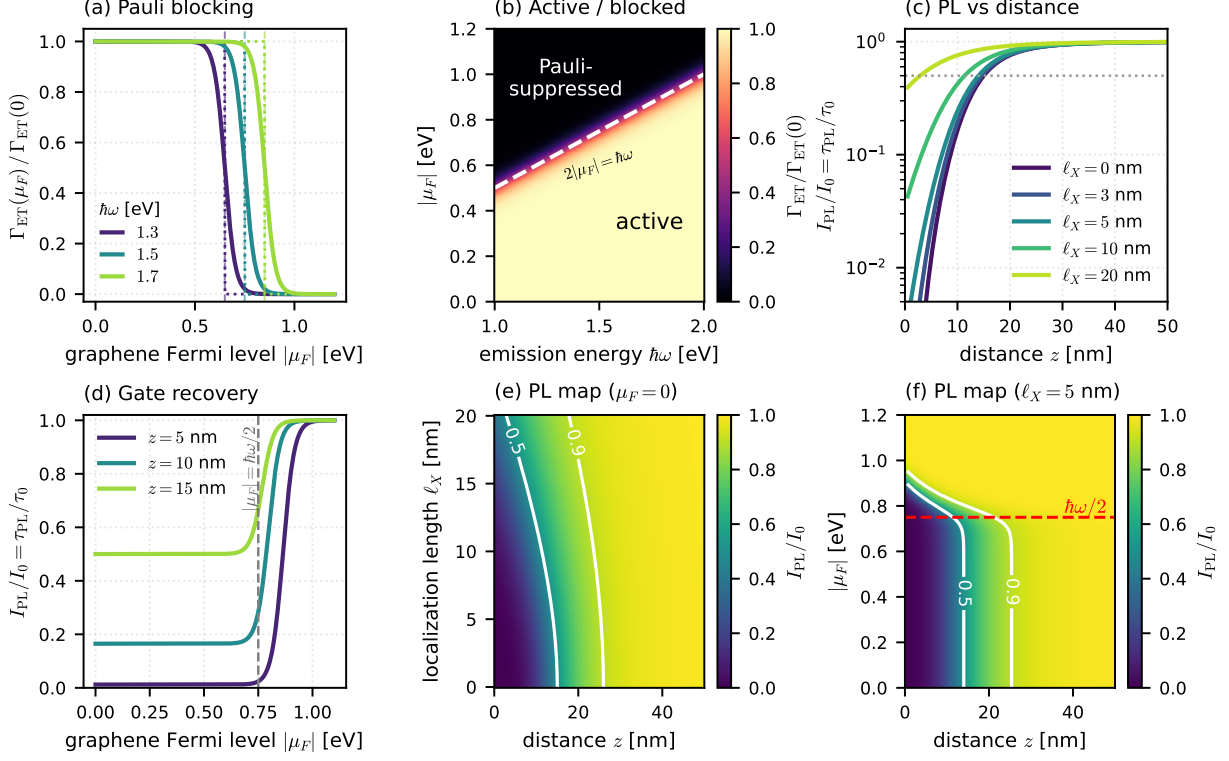


FIG. 4. (Color online) Gate tunability and PL observables. (a) ET suppression factor versus $|\mu_F|$ for three emission energies (solid: $T = 300$ K; dotted: $T = 0$ step; vertical dashed: $|\mu_F| = \hbar\omega/2$); (b) active versus Pauli-blocked regions in the $(\hbar\omega, |\mu_F|)$ plane; (c) relative PL $I_{\text{PL}}/I_0 = \tau_{\text{PL}}/\tau_0$ versus distance (log scale) for several ℓ_X at $\mu_F = 0$ (fixed-dipole benchmark, for which $R_X = \ell_X$); (d) relative PL versus Fermi level for a point dipole ($\ell_X = 0$) at several fixed distances, showing gate-induced recovery; (e) PL map in (z, ℓ_X) at $\mu_F = 0$ (fixed-dipole benchmark); (f) PL map in $(z, |\mu_F|)$ at $\ell_X = 5$ nm (dashed: $|\mu_F| = \hbar\omega/2$; contours 0.5, 0.9). Panels (b), (d), and (f) use the sharp $q = 0$ optical-limit blocking factor; the quantitative finite- q gate dependence is given by the full RPA in Fig. 5.

D. PL observables and parameter regimes

Translating to observables through Eq. (12), Fig. 4(c) shows the relative PL versus distance: at $z = 5$ nm and $\mu_F = 0$ a point-dipole emitter is strongly quenched ($I_{\text{PL}}/I_0 \approx 0.01$), while a finite exciton recovers—for the fixed-dipole benchmark with $\ell_X = 10$ nm, to ≈ 0.16 —because its form factor suppresses the short-wavelength (high- q) near field that couples it to graphene. Figure 4(d) shows the same ratio versus gate voltage: gating graphene past

$|\mu_F| = \hbar\omega/2$ partially restores the PL even at fixed distance, an independent electrical means of distinguishing graphene-induced quenching from disorder.

Figure 4(e,f) maps the relative PL I_{PL}/I_0 over the device parameter space. In the (z, ℓ_X) plane [panel (e)], the contour $I_{\text{PL}}/I_0 = 0.9$ at which quenching becomes negligible lies near $z \approx 24\text{--}26$ nm and depends only weakly on ℓ_X , while strong quenching (< 0.5) dominates below $z \sim 10\text{--}15$ nm. In the $(z, |\mu_F|)$ plane [panel (f)], gating above $\hbar\omega/2$ suppresses the quenching, most completely at the larger spacings; the quantitative, momentum-resolved gate requirement follows from the full RPA [Fig. 5(c,d)]. These maps delineate when proximity-induced ET must be included in the interpretation of moiré-exciton PL.

One caveat on reading the ℓ_X -varying panels [(c),(e)]: they are drawn with the benchmark, whose ℓ_X -independent Γ_0 lets one d_0 normalize the whole family, whereas the fixed- ℓ_X panels [(b),(d),(f)] carry D_X only as a constant and are model-independent. For the physical coherent exciton the giant oscillator strength puts $|D_X|^2 \propto \ell_X^2$ [Eq. (4)] into *both* Γ_{rad} and Γ_{ET} . For a radiatively limited emitter ($\Gamma_0 \simeq \Gamma_{\text{rad}} \propto \ell_X^2$) this common factor cancels in $x = \Gamma_{\text{ET}}/\Gamma_0$; the benchmark curves then apply once the size is expressed through the transition-polarization radius R_X , since $R_X = \sqrt{2}\ell_X$ for the coherent Gaussian but $R_X = \ell_X$ for the benchmark. If instead Γ_{nr}^0 dominates (Γ_0 nearly ℓ_X -independent), an additional ℓ_X^2 dependence remains in the absolute quenching level, while the normalized momentum-filtering crossover is still governed by R_X .

Worked example. To make these trends concrete, consider a TMDC heterobilayer with a moiré exciton of transition-polarization radius $R_X = 3$ nm separated from a graphene gate by an hBN spacer, taking $\hbar\omega = 1.5$ eV and the representative $d_0 = 15$ nm. At $z = 5$ nm and $\mu_F = 0$ a point emitter is quenched to $I_{\text{PL}}/I_0 \approx 0.01$, whereas the finite form factor lifts this to ≈ 0.02 for $R_X = 3$ nm and ≈ 0.04 for $R_X = 5$ nm (fixed-dipole benchmark), so the quenching is strong but measurably size dependent. The finite-size fingerprint is read most directly from the deviation of the rate from the point-dipole response evaluated with the same bath [Fig. 3(d)]. For $R_X = 3$ nm this deviation is $\approx 48\%$ at $z = 5$ nm and $\approx 18\%$ at $z = 10$ nm, and for $R_X = 5$ nm it is $\approx 73\%$ and $\approx 38\%$ at the same distances. A spacer-thickness lifetime series resolves this variation and is fit for R_X with the full Gaussian form factor. Equation (8) alone suffices only at the larger- z end ($z/R_X \gtrsim 4\text{--}5$), so the $R_X = 5$ nm case here ($z/R_X = 1\text{--}2$) lies in the crossover and requires the full kernel. (These illustrative numbers are isotropic; the anisotropic hBN spacer replaces z by $z_{\text{eff}} = \gamma z$.)

Raising the gate to $|\mu_F| = 0.9$ eV, past the threshold $\hbar\omega/2 = 0.75$ eV, suppresses the interband-dominated full-RPA ET rate. Evaluated with the $R_X = 3$ nm form factor (not shown) this suppression reaches $\approx 99.5\%$ at $z = 5$ nm, lifting I_{PL}/I_0 from ≈ 0.02 to ≈ 0.8 . The point-source gate sweep of Fig. 5(c) recovers somewhat less ($\approx 97\%$ suppression at $z = 5$ nm), its higher- q weight being harder to Pauli-block. Both are $T = 0$ estimates. Thermal smearing at 300 K would reduce the ET suppression and hence the PL recovery. The recovery is moreover regime dependent, incomplete in the strongest-quenching region ($z \lesssim 4$ nm, $x \gtrsim 80$) but near-complete at intermediate spacings ($z \sim 10$ – 15 nm, $x \sim 1$ – 5). All three signatures—strong quenching, its size dependence, and gate recovery—are experimentally testable, though the required $|\mu_F| \sim 0.75$ – 0.9 eV ($n \sim 4$ – 6×10^{13} cm $^{-2}$) is most accessible with ionic or dual-gate techniques or for the lower-energy interlayer excitons ($|\mu_F| \sim 0.55$ – 0.7 eV).

E. Beyond the minimal model: full RPA loss function

To test the minimal model we return to the general overlap formula Eq. (3) and evaluate the graphene density response χ_G within the RPA, replacing its small- q neutral form $-\text{Im} \chi_G \propto q^2/\omega$ [Eq. (5)] by the full loss function of doped graphene, as also used for moiré-exciton quenching in Ref. [26]. The non-interacting polarizability $\Pi(q, \omega)$ at $T = 0$ is known in closed form [27, 43, 44]. We use their convention, in which $\Pi(q \rightarrow 0, 0) = D_0 = gE_F/2\pi\hbar^2v_F^2 > 0$ ($g = 4$, with $E_F = |\mu_F|$ the Fermi energy and $k_F = E_F/\hbar v_F$) and the RPA dielectric function is $\varepsilon(q, \omega) = 1 + v_q\Pi$ with $v_q = 2\pi e^2/(\kappa q)$. Within this graphene RPA screening function the surrounding dielectric—here the hBN encapsulation—enters through κ in v_q (the average permittivity of the media on either side of the sheet) [44], while the bare bubble Π is a property of the graphene alone; the separate effect of the environment on propagation from the emitter to graphene is contained in $T(q, \omega)$ and in the spacer-anisotropy treatment of Sec. III F. The screened density response is $\chi_{\text{RPA}} = -\Pi/\varepsilon$ and the loss function is

$$L(q, \omega, \mu_F) = -\text{Im} \chi_{\text{RPA}} = \text{Im} \left[\frac{\Pi(q, \omega)}{\varepsilon(q, \omega)} \right], \quad (13)$$

which contains the intraband and interband single-particle continua, their Pauli-blocked phase space at finite μ_F , and the screened plasmon.

Since the emitter couples through its *bare* near-field potential ϕ_X [Eq. (3)], the relevant

kernel is the screened density response $-\text{Im} \chi_{\text{RPA}} = \text{Im}[\Pi/\varepsilon]$, not the energy-loss function $-\text{Im}(1/\varepsilon)$. The two are not independent. Since $1/\varepsilon = 1 + v_q \chi_{\text{RPA}}$, one has $-\text{Im}(1/\varepsilon) = v_q (-\text{Im} \chi_{\text{RPA}})$, so an equivalent formulation using $-\text{Im}(1/\varepsilon)$ would move this factor of $v_q \propto 1/q$ from the response kernel into the source coupling. The transfer rate is then

$$\Gamma_{\text{ET}} \propto |D_X(\ell_X)|^2 \int_0^\infty q dq e^{-2qz} |F_X(q; \ell_X)|^2 L(q, \omega, \mu_F), \quad (14)$$

with $|F_X|^2$ as in Eq. (4).

We verified our implementation of Π against the closed-form static limit [45], $\Pi(q, 0) = D_0$ for $q < 2k_F$, the positivity of $\text{Im} \Pi$ in this convention (so that $-\text{Im} \chi_{\text{RPA}} > 0$ is the energy loss), the neutral-graphene interband form $\text{Im} \Pi = q^2/(4\sqrt{\omega^2 - v_F^2 q^2})$, and the existence of a $\sqrt{q}$ plasmon. The polarizability is evaluated at $T = 0$. The rate integrals use $\eta = 0$, with the integrable square-root singularities at the kinematic edges handled by adaptive quadrature with breakpoints there and a momentum cutoff where e^{-2qz} is negligible. Tightening the adaptive-quadrature tolerance and extending the momentum cutoff leave the rate integrals and the qualitative scale separation unchanged. The RPA/minimal comparison is of the normalized distance shape (relative to $z = 20$ nm), not the absolute rate. The polarizability is the linear-Dirac-cone RPA. At the highest Fermi levels considered ($|\mu_F| \sim 0.9$ eV) its assumptions—a linear dispersion, electron–hole symmetry, and constant v_F —are least accurate, so lattice-band (trigonal-warping) corrections [36] and the measured optical conductivity may be needed for quantitative modeling there, though the qualitative gate dependence is unchanged.

Figure 5(a) shows the loss function for hBN-encapsulated graphene ($\kappa = 4.5$, $\alpha = e^2/\kappa \hbar v_F \approx 0.49$). This representative value agrees to within $\sim 15\%$ with the optical $\kappa_{\text{eff}} = \sqrt{\varepsilon_{\parallel} \varepsilon_{\perp}} \simeq 3.9$ from Sec. III F, and the small- q result below is independent of κ since $\varepsilon \rightarrow 1$ there. At the exciton emission energy $\hbar\omega = 1.5$ eV $\gg E_F$, the response is interband-dominated. At small q —the only momenta that survive the e^{-2qz} near-field filter at experimentally relevant z —the screened loss function converges to the neutral-graphene interband result, because $\text{Re} \Pi \rightarrow 0$ there and $\varepsilon \rightarrow 1$. Doping (Pauli blocking of part of the interband phase space) and screening only suppress L at intermediate q . Consequently the full-RPA rate follows the z^{-4} law at large z [Fig. 5(b), fitted slope -3.998] and, for the representative parameters there, its normalized distance dependence agrees with the minimal model to within a few percent for $z \gtrsim 3$ nm, deviating by $\lesssim 10\%$ only at $z \approx 2$ nm.

The minimal model Eq. (5) is therefore quantitatively reliable for the distance scaling in the regime of interest, and genuine Fermi-level control of the rate at this emission energy requires reaching the Pauli threshold $|\mu_F| > \hbar\omega/2$, consistent with Sec. III above.

The RPA also validates the gate dependence [Fig. 5(c,d)]. The suppression with $|\mu_F|$ is gradual and z -dependent, and near-complete blocking of the near-field momenta $q \sim q_{\text{peak}} = 3/2z$ requires $|\mu_F|$ rising from ≈ 0.75 eV at large z to ≈ 0.85 – 0.91 eV at $z = 5$ – 3 nm. This tracks the analytic estimate $(\hbar\omega + \hbar v_F q_{\text{peak}})/2$ and confirms that the sharp $q = 0$ model underestimates the gate needed at short range.

F. Anisotropic hBN spacer

The dielectric environment is known to influence moiré-exciton energy transfer to graphene [26]. Here we make the role of the spacer anisotropy explicit. The hBN spacer is a uniaxial dielectric with in-plane permittivity $\varepsilon_{\parallel}$ and out-of-plane $\varepsilon_{\perp}$. Solving the anisotropic Laplace equation $\varepsilon_{\parallel} q^2 \phi = \varepsilon_{\perp} \phi''$ for an in-plane mode q shows that the near field decays vertically as $e^{-\gamma q z}$ with

$$\gamma = \sqrt{\varepsilon_{\parallel}/\varepsilon_{\perp}}, \quad (15)$$

so, for the in-plane optical transition dipole relevant to TMDC excitons, anisotropy enters the rate only through the replacement $z \rightarrow \gamma z$ in the near-field factor (a perpendicular dipole would acquire an additional γ prefactor from the vertical derivative, but the in-plane form factor is unchanged), while the coupling is screened by $\kappa_{\text{eff}} = \sqrt{\varepsilon_{\parallel}\varepsilon_{\perp}}$, taken in the local-dielectric limit where κ_{eff} is q -independent.

For bulk hBN the relevant response at the optical transition energy $\hbar\omega = 1.5$ eV is the electronic (high-frequency) permittivity, $\varepsilon_{\parallel} \simeq 4.98$ and $\varepsilon_{\perp} \simeq 3.03$ [46], giving $\gamma \simeq 1.28$ and $\kappa_{\text{eff}} \simeq 3.9$. The static values $\varepsilon_{\parallel}^0 \simeq 6.93$ and $\varepsilon_{\perp}^0 \simeq 3.76$ include the ionic contribution and apply only below the hBN phonon bands, not at the exciton energy. As shown in Fig. 6(a,b), the z^{-4} rate is then suppressed by $\gamma^4 \simeq 2.7$ relative to an isotropic treatment, and the graphene quenching distance shrinks by $1/\gamma \simeq 0.78$ (a $\sim 28\%$ overestimate of the range if anisotropy is neglected).

The anisotropy also propagates into the size extraction. With $z \rightarrow \gamma z$, Eq. (8) becomes $\Gamma_{\text{ET}} \propto (\gamma z)^{-4} [1 - \frac{5}{2} R_X^2 / (\gamma z)^2 + \dots]$ and the half-suppression sits at $\gamma z \simeq 1.6 R_X$, so a fit using the physical thickness z in place of the effective thickness $z_{\text{eff}} = \gamma z$ underestimates

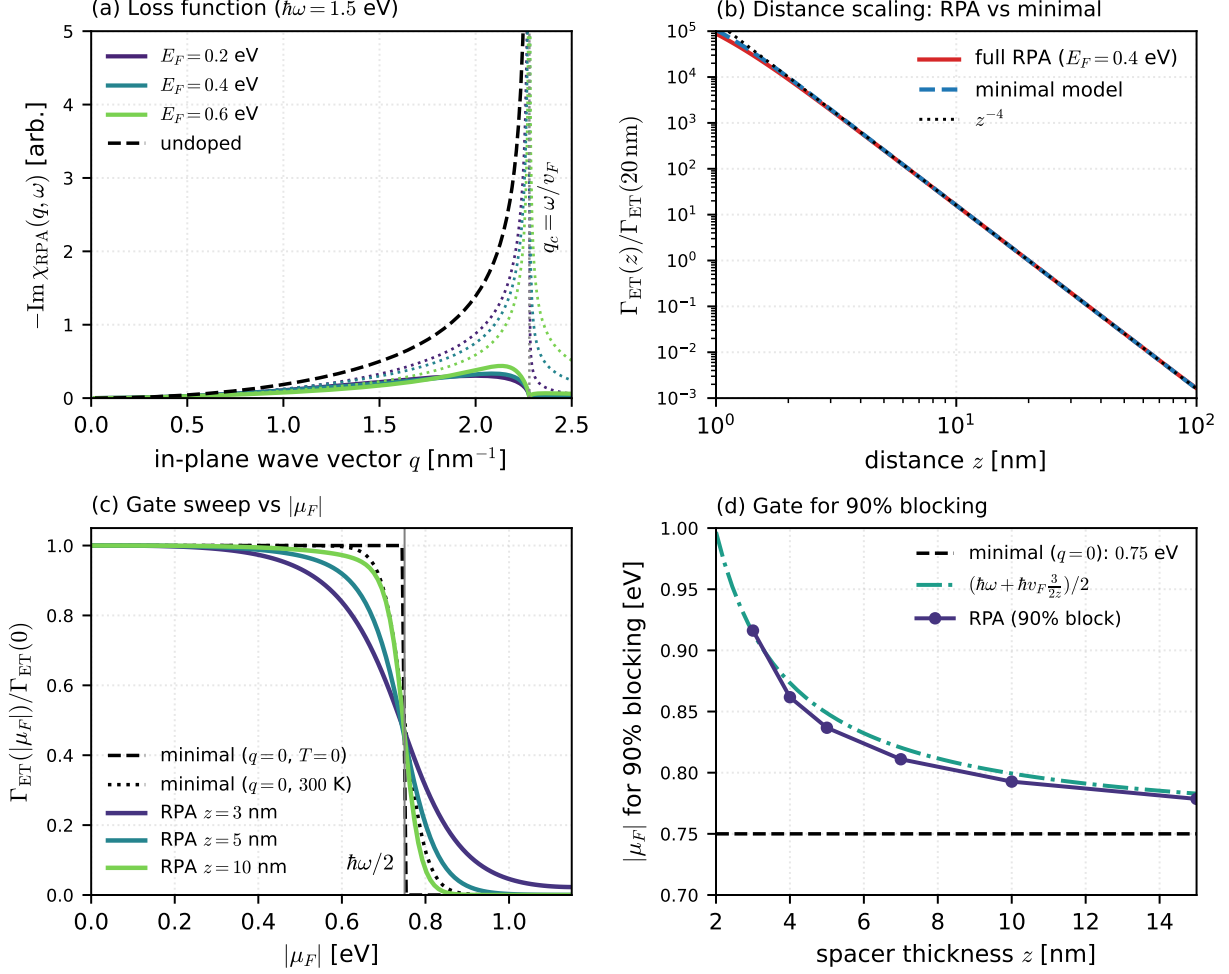


FIG. 5. (Color online) Full-RPA validation of the minimal model, for distance and gate dependence. (a) $L(q, \omega)$ at $\hbar\omega = 1.5$ eV for several Fermi levels (solid: screened RPA; dotted: bare bubble; dashed: neutral-graphene interband reference). (b) $\Gamma_{\text{ET}}(z)$ from the RPA loss (solid) versus the minimal model (dashed), each normalized at $z = 20$ nm; both follow z^{-4} (dotted), deviating only at small z . (c) Gate sweep: ET suppression $\Gamma_{\text{ET}}(|\mu_F|)/\Gamma_{\text{ET}}(0)$ versus Fermi level from the full RPA at $z = 3, 5, 10$ nm (solid), compared with the sharp $q = 0$ minimal model (dashed $T = 0$ step, dotted 300 K). The RPA suppression is gradual and z -dependent, with a high- q tail extending past the $q = 0$ threshold $\hbar\omega/2 = 0.75$ eV (vertical line). (d) Gate needed for 90% blocking versus spacer thickness: the RPA (points) tracks the estimate $(\hbar\omega + \hbar v_F q_{\text{peak}})/2$ with $q_{\text{peak}} = 3/2z$ (dash-dotted) and rises well above the $q = 0$ value (0.75 eV, dashed), reaching ≈ 0.85 – 0.91 eV at $z = 5$ – 3 nm. Panels (c,d) use the point-source form factor ($F_X = 1$); a finite exciton size shifts the near-field weight to lower q , so the gate needed for a given blocking level is somewhat lower, making these curves an upper estimate for an extended source.

R_X by $1/\gamma$ ($\sim 22\%$ for bulk hBN). The effective thickness $z_{\text{eff}} = \gamma z$ should therefore be used when applying Eq. (8) and the crossover of Fig. 3(d) to extract R_X from a measured spacer-thickness series. When d_0 is itself calibrated in the same anisotropic environment, the overall γ^{-4} suppression of the point-dipole rate is absorbed into d_0 and must not be applied again; only the finite-size crossover, set by $F_X(q)$ at $q \sim 1/\gamma z$, then carries the effective thickness.

G. Plasmon-resonant regime and validity boundary

Besides the interband continuum, the RPA loss function (13) hosts a $\sqrt{q}$ plasmon pole in the single-particle gap [Fig. 6(c), rendered with a small broadening $\eta = 0.05 E_F$], the basic collective mode of graphene plasmonics [47, 48]. Energy transfer is resonantly enhanced—and deviates from the z^{-4} law—when the emission energy matches the plasmon at the momenta admitted by the near-field filter e^{-2qz} . That filter has exponential cutoff scale $q \sim 1/2z$, whereas the rate integrand $q^3 e^{-2qz}$ peaks at the larger $q_{\text{peak}} = 3/2z$ (with mean $\langle q \rangle = 2/z$). We take this integrand peak as the representative near-field momentum, so that $\hbar\omega \simeq \hbar\omega_{\text{pl}}(q_{\text{peak}})$. These momentum scales ($1/2z$, $1/z$, $3/2z$) differ only by numerical factors of order unity and all track the inverse spacer thickness. Graphene plasmons are, however, mid-infrared. At $E_F = 0.4$ eV the resonant energy $\omega_{\text{res}}(z) = \omega_{\text{pl}}(3/2z)$ ranges only ~ 140 – 310 meV for $z = 20$ – 5 nm [Fig. 6(d)], still well below TMDC exciton energies (1.1–1.8 eV). The plasmon channel is therefore off-resonant for moiré excitons—which lie well within the interband regime where the minimal model holds—and would dominate only for infrared emitters or much higher doping. This scale separation delimits the validity of the interband-based theory. For visible/near-infrared moiré-exciton PL, the collective mode can be neglected and Eq. (5) applies. Coupling to the plasmon becomes a distinct, distance-tunable resonance only in the infrared.

Throughout we use the non-retarded (electrostatic) near field and the correspondingly non-retarded $\sqrt{q}$ plasmon dispersion. This is justified because, for the nanometric strong-quenching regime emphasized here, the representative near-field momentum $q_{\text{peak}} = 3/2z$ lies well above the light line ω/c , where retardation would modify the dispersion. Retardation becomes important only at much longer wavelengths, where it drives a crossover from linear to sublinear plasmon dispersion in graphene [49]. In the mid-infrared the hBN dielectric

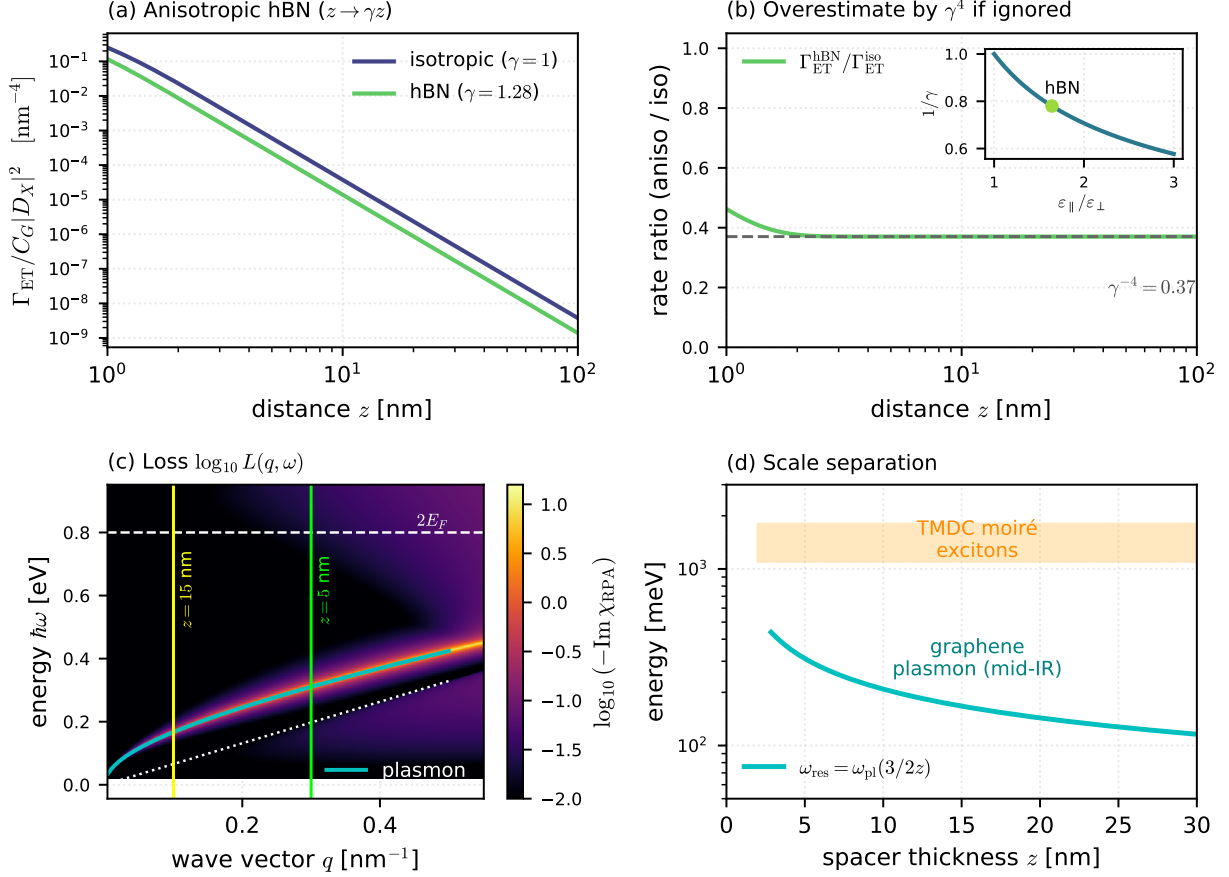


FIG. 6. (Color online) Validity checks of the interband minimal model. Top row (anisotropic hBN spacer): (a) point-dipole $\Gamma_{\text{ET}}(z)$ for an isotropic spacer ($\gamma = 1$) and bulk hBN ($\gamma = 1.28$; anisotropy acts as $z \rightarrow \gamma z$); (b) the rate ratio plateaus at $\gamma^{-4} \simeq 0.37$ at large z , inset the quenching-distance factor $1/\gamma$ versus $\varepsilon_{\parallel}/\varepsilon_{\perp}$ with bulk hBN marked. Bottom row (plasmon regime): (c) RPA loss $\log_{10} L(q, \omega)$ at $E_F = 0.4$ eV showing the $\sqrt{q}$ plasmon ridge below the interband region, with the representative near-field momentum $q \simeq 3/2z$ (the rate-integrand peak) for $z = 5, 15$ nm (dotted line: $\omega = v_F q$; dashed: $2E_F$); (d) the resonant energy $\omega_{\text{pl}}(3/2z)$ versus spacer thickness lies in the mid-infrared, well separated from the TMDC moiré-exciton band.

response is itself strongly frequency-dependent, and the graphene plasmon can hybridize with hBN phonon-polaritons [50]. Figure 6(d) is therefore a bare-graphene order-of-magnitude estimate, which does not affect the visible/near-infrared conclusion.

IV. DISCUSSION

The theory above is deliberately minimal, and a few assumptions delimit its scope. First, the Gaussian form factor (4) is the simplest localized-exciton model: a realistic moiré wave function alters the detailed crossover through higher moments of $F_X(q)$, but the leading long-distance deviation—and hence the extracted R_X —is unchanged. Second, the coherent scaling $|D_X|^2 \propto \ell_X^2$ (the giant oscillator strength) is an idealization. It holds only in weak confinement, where the center-of-mass envelope ℓ_X exceeds the internal exciton radius $a_X \sim 1\text{--}2$ nm (the electron–hole relative-coordinate size); the smallest ℓ_X plotted in Fig. 3 fall below this bound and are included only to illustrate the size dependence, not as physical cases. A coherent exciton radiates like a phased antenna array: the unit-cell transition dipoles add *in phase*, so the bright-state $|D_X|^2$ grows with the coherent area. This in-phase addition breaks down beyond the phase-coherence length L_ϕ , the moiré-domain size L_{domain} , or the optical wavelength, so $|D_X|^2$ saturates at $\min(\ell_X^2, L_\phi^2, L_{\text{domain}}^2, A_{\text{opt}})$ rather than growing without limit—equivalently, the oscillator-strength sum rule conserves the total and only redistributes it. Real excitons are only partially coherent—phonons and disorder degrade the coherence—so $|D_X|^2$ grows with ℓ_X more weakly than the ideal ℓ_X^2 law. Their absolute size dependence therefore lies between the coherent and fixed-dipole curves of Fig. 3.

The bath and the emitter contribute in distinct distance regimes. The intermediate-distance z^{-4} asymptote reflects the small- q response of the graphene bath, independent of the emitter’s internal structure. It is therefore recovered for any localized emitter—a molecule or an exciton wave packet—once z exceeds its coherence length, within the nonretarded window $R_X, q_c^{-1} \ll z \ll c/\omega \simeq 130$ nm at $\hbar\omega = 1.5$ eV. By contrast, an ideal free exciton—translationally invariant, with a sharp center-of-mass momentum Q so that $P_X(q) \propto \delta(q - Q)$ —is a distinct limit. Its rate samples the bath at that single momentum, $\propto e^{-2Qz}[-\text{Im}\chi_G(Q, \omega)]$, and vanishes as Q^2 for a bright ($Q \rightarrow 0$) state, since $-\text{Im}\chi_G \propto q^2$. A delocalized bright exciton therefore barely couples to graphene: it is the finite- q spread of a *localized* wave packet that provides the coupling. The finite-size deviation and its R_X dependence are thus where the tunable moiré confinement is essential.

The same formalism applies to disorder-localized monolayer excitons (with ℓ_X replaced by a random ℓ_{dis}). The advantage of moiré excitons is that ℓ_X is set by twist angle and moiré

depth rather than by uncontrolled disorder. In practice both contributions coexist, and the resulting spatial inhomogeneity of moiré-exciton PL can be mapped and statistically disentangled by hyperspectral imaging and descriptor-correlation analyses [18, 51]. The theory should thus be read as a general exciton-to-graphene energy-transfer formalism, with moiré excitons providing the most controlled realization of a finite and tunable emitter form factor.

The same reasoning indicates what changes if the graphene gate is replaced by multilayer graphene or graphite, as in many real devices. The z^{-4} law is specific to a single 2D sheet. An independent-layer sum over the interlayer spacing $d \simeq 0.34$ nm gives $\sum_{n \geq 0} \Gamma_{\text{ET}}^{(1)}(z+nd) \sim \int_z^\infty dz'/z'^4 \sim 1/(3dz^3)$ for $z \gg d$, i.e. a crossover toward $\Gamma_{\text{ET}} \propto z^{-3}$ —the surface-energy-transfer law for a molecule near an absorbing half-space [52]. Because z^{-3} falls off more slowly than z^{-4} , graphite quenches more strongly and over a longer range (by a factor $\sim z/3d \sim 5$ at $z = 5$ nm). The emitter-side sensitivity to R_X persists, but the softer $\sum_n e^{-2q(z+nd)} \simeq e^{-2qz}/(2qd)$ kernel lowers the leading correction coefficient from $5/2$ to $\simeq 3/2$. Gate tunability is largely lost, because c -axis screening confines doping to the top one or two layers and Pauli blocking then removes only a fraction $\sim d/z$ of the response. An electrically switchable bath is thus specific to monolayer (or few-layer) graphene, whereas graphite is a permanently active, longer-ranged quencher. This independent-layer picture is a scaling argument rather than a quantitative prediction, and a realistic treatment would use the graphite bulk and surface dielectric response.

The sharp Pauli-blocking factor (10) omits the intraband (Drude) and plasmonic contributions of the full RPA polarizability. As shown in Secs. III E and III G, at $\hbar\omega \gg E_F$ these are confined to intermediate momenta or to the mid-infrared, leaving the minimal model accurate for visible/near-infrared excitons at $z \gtrsim 3$ nm. Finite temperature and hBN anisotropy [Eqs. (11) and (15)] likewise fit within the overlap form (3) without altering its structure.

A further channel not treated here is direct interlayer charge transfer, which can compete with energy transfer in directly contacted graphene–TMDC stacks [53]. The present theory addresses the nonradiative *energy*-transfer pathway, which dominates once a few nanometers of hBN suppress the wave function overlap required for tunneling while leaving the long-range Coulomb coupling intact. Consistent with this picture, a recent distance-resolved study of WS₂/graphene found Förster-type energy transfer to dominate at finite spacer thickness,

with a Dexter (tunneling) contribution only at direct contact [54]. Energy transfer that quenches and spectrally filters the photoluminescence of an atomically thin semiconductor placed next to graphene has indeed been observed directly [55], and rapid interlayer energy transfer between dissimilar TMDC monolayers is likewise well established experimentally [56] and theoretically [57].

Two features make the predictions experimentally testable. First, because the relative PL intensity and lifetime are predicted to track each other [Eq. (12)], a measured correlation between brightness and lifetime as a function of spacer thickness is a diagnostic signature of ET rather than of varying disorder, especially when combined with gate tuning. In practice I_{PL} is also affected by absorption of the excitation light, collection efficiency, exciton diffusion, and trap capture, so the equality $I_{\text{PL}}/I_0 = \tau_{\text{PL}}/\tau_0$ should be read as a diagnostic limit in which only Γ_{ET} varies. Lifetime measurements are insensitive to excitation absorption and collection efficiency and therefore provide the cleaner test, although diffusion, trap capture, and state feeding can still modify the decay dynamics and must be controlled. Second, the gate dependence provides an *in situ* control. The same device can be driven between the active and Pauli-blocked regimes, and the progressive PL recovery across this transition isolates the graphene contribution. The required $|\mu_F| \sim 0.6\text{--}0.9$ eV, depending on emission energy and spacer thickness, is demanding for conventional hBN-gated devices at visible exciton energies, but becomes more accessible for the lower-energy interlayer excitons ($\hbar\omega \sim 1.1\text{--}1.4$ eV, $|\mu_F| \sim 0.55\text{--}0.7$ eV), for infrared emitters, or for devices using strong electrostatic or ionic gating. Gate control of interlayer-exciton dynamics in such devices is already established [58]. The universal distance scaling and gate control already demonstrated for point emitters [4, 7] support the underlying mechanism. The new, testable element here is the dependence on the moiré-exciton transition-polarization radius R_X .

A concrete protocol makes the extraction well posed and circumvents the phenomenological normalization. Because the graphene gate strongly modulates the nonradiative channel between active and approximately Pauli-blocked states, one can measure the time-resolved PL of the *same spectroscopically identified exciton family* at each spacer thickness z in both the active ($\mu_F = 0$) and Pauli-blocked ($|\mu_F| > \hbar\omega/2$) states and form the difference

$$\Delta\Gamma_{\text{ET}}(z) \equiv \frac{1}{\tau_{\text{active}}(z)} - \frac{1}{\tau_{\text{blocked}}(z)}. \quad (16)$$

Only Γ_{ET} responds to the gate; the radiative rate, the intrinsic nonradiative rate, and

any disorder and diffusion losses are gate-independent, hence identical in the active and blocked states, and cancel in the difference—leaving the change in Γ_{ET} . The subtraction is not exact. Because the interband threshold is q -dependent, the blocked state keeps a small finite- q interband tail, so $\Delta\Gamma_{\text{ET}}$ equals Γ_{ET} only up to this residual, $\Delta\Gamma_{\text{ET}}(z) \propto |D_X|^2 \int q dq e^{-2qz} |F_X(q)|^2 [L(q, \omega, \mu_{\text{active}}) - L(q, \omega, \mu_{\text{blocked}})]$, most pronounced at short spacings where $q_{\text{peak}} = 3/2z$ is largest. The simple leading-order fit [Eq. (8)] is therefore rigorous only for complete blocking; in general one extracts R_X by fitting against the full-RPA difference kernel $L(q, \omega, \mu_{\text{active}}) - L(q, \omega, \mu_{\text{blocked}})$ [Sec. III E, Fig. 5(c,d)].

The subtraction isolates Γ_{ET} only if gating the graphene leaves the exciton itself unchanged; otherwise the active and blocked states differ by more than the loss channel. Two effects must be avoided: the gate field Stark-shifting the exciton energy, and the changed graphene screening renormalizing D_X , R_X , or ω_X . Both are suppressed by holding the exciton energy, charge state, and displacement field fixed, ideally through dual-gate compensation. This is checked from the exciton energy, linewidth, polarization, and radiative spectral weight remaining unchanged. The selected line should also decay approximately single-exponentially. If it does not, the active and blocked transients must be fit together with a shared multilevel rate-equation model, since graphene also changes the lifetimes of the states that feed the line, not only that of the line itself.

Taking the ratio $\Delta\Gamma_{\text{ET}}(z)/\Delta\Gamma_{\text{ET}}(z_{\text{ref}})$ to a reference thickness then removes the unknown oscillator strength $|D_X|^2$, the absolute prefactor C_G , and d_0 . The remaining normalized distance dependence is fit for R_X against the known graphene kernel and the effective thickness $z_{\text{eff}} = \gamma z$ [Eq. (8) and Secs. III B, III F], not against a bare z^{-4} law. Here z is the distance from the exciton transition-polarization centroid to the graphene plane, not the nominal hBN thickness. Restricting the sweep to $z \gtrsim 2\text{--}3$ nm keeps the measurement in the Förster (energy-transfer) regime, away from the Dexter/charge-transfer channel at direct contact [53, 54]. The measurement sensitivity, the parameter correlations that set the uncertainty on R_X , the spacer offset, and the device-to-device statistics required by a multi-thickness series are collected in Appendix C. Being both differential and self-referenced, the scheme extracts R_X from the deviation of the finite-size signal from the calibrated point-source difference kernel, without recourse to an absolute rate.

For the same reason, the PL maps of Fig. 4(e,f) should be read as scaling illustrations. They use the fixed-dipole benchmark and the representative d_0 . The RPA comparison

validates the graphene loss function and the distance dependence of the kernel, but not the absolute exciton transition source or prefactor.

V. CONCLUSION

We have formulated nonradiative energy transfer from moiré excitons to graphene as the overlap between the exciton near-field spectrum and the long-wavelength electronic density response of graphene, weighted by an exciton form factor. The framework reproduces the known z^{-4} law in the point-dipole limit. Its leading deviation from that law, measured relative to the calibrated point-dipole response, is governed at long distance by the transition-polarization radius R_X and thus probes the moiré-exciton extent. In the ideal fully coherent limit the size dependence is non-monotonic, peaking at $\ell_X \approx z$. Finally, Pauli blocking makes the nonradiative channel electrically tunable. The cleanest experimental test is a gate-referenced, spacer-thickness-dependent lifetime measurement. Its active-blocked difference isolates the graphene-induced change in the decay rate; the deviation of its normalized distance dependence from the calibrated point-dipole response then yields R_X at long distance. With an assumed transition-polarization profile, the full crossover can be used as well. Gate tuning of the bath is most favorable for low-energy interlayer excitons and, more generally, for infrared emitters, where the Pauli threshold $|\mu_F| \sim \hbar\omega/2$ and the graphene plasmon are most accessible. In van der Waals exciton devices, a graphene gate should therefore be regarded not as a passive electrostatic element but as a tunable 2D nonradiative reservoir whose long-wavelength response can be probed through exciton photoluminescence quenching.

ACKNOWLEDGMENTS

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Appendix A: Neutral-graphene response and the q^3 kernel

The minimal kernel of Eq. (5) follows from the standard small- q , Dirac-cone density response of neutral graphene. At $T = 0$ and for $\omega > v_F q$, the imaginary part of the noninteracting polarizability is

$$-\text{Im } \chi_0(q, \omega) = \frac{g}{16} \frac{q^2}{\sqrt{\omega^2 - v_F^2 q^2}} \Theta(\omega - v_F q), \quad (\text{A1})$$

with degeneracy $g = 4$ (we set $\hbar = 1$ in this Appendix). The step $\Theta(\omega - v_F q)$ is kinematic. A Dirac interband pair of total energy ω carrying momentum q requires $|\mathbf{k}| + |\mathbf{k} + \mathbf{q}| = \omega/v_F$, which by the triangle inequality is $\geq q$, so no such pair—and hence no absorption—exists above the cutoff $q_c = \omega/v_F$. The same Dirac-cone polarization, evaluated statically, gives the q -linear interband result $\Pi^0(q) = g q/(16v_F)$ [45], sharing the $g/16$ prefactor. For $\omega \gg v_F q$ —the regime selected by the near-field filter at experimentally relevant z —this reduces to $-\text{Im } \chi_0 \simeq (g/16) q^2/\omega \propto q^2/\omega$, the form used in the main text. This long-wavelength interband response is the same physics as the frequency-independent universal optical conductance $\sigma_0 = \pi e^2/2h$ of graphene, predicted by Ando *et al.* [59] and measured in Ref. [60], here resolved at finite q .

A transition dipole at height z couples to graphene through its bare Coulomb potential. For a dipole perpendicular to the plane, the in-plane Fourier component of the potential is $\phi_X(q, z) \propto e^{-qz}$ (the dipole derivative ∂_z brings a factor q that cancels the $1/q$ of the 2D Coulomb kernel $2\pi/q$), so $|\phi_X|^2 \propto e^{-2qz}$. An in-plane dipole gives $|\phi_X|^2 \propto \cos^2\theta e^{-2qz}$, whose azimuthal average is $\frac{1}{2}e^{-2qz}$. The orientation enters only through this constant, absorbed into $C_G(\omega)$. Inserting both factors into the overlap form Eq. (3) with the planar measure $d^2q = 2\pi q dq$ gives

$$\Gamma_{\text{ET}} \propto \int q dq \underbrace{e^{-2qz}}_{|\phi_X|^2} \underbrace{\frac{q^2}{\omega}}_{-\text{Im } \chi_0} = \frac{1}{\omega} \int dq q^3 e^{-2qz}, \quad (\text{A2})$$

i.e. Eq. (5) with $C_G(\omega) \propto 1/\omega$, and $\int_0^\infty q^3 e^{-2qz} dq = 3/(8z^4)$ reproduces the Swathi–Sebastian z^{-4} law [1, 2]. The chirality factor of the Dirac matrix elements is contained in Eq. (A1) and likewise affects only the prefactor, not the z -dependence.

Appendix B: Gaussian exciton form factor

The optical transition polarization of an exciton is linear in its center-of-mass amplitude. Coarse-grained over the envelope scale, $\langle 0 | \hat{\mathbf{P}}(\mathbf{R}) | X \rangle \propto \psi_X(\mathbf{R})$, with the microscopic inter-band dipole and the relative-coordinate wave function at zero electron-hole separation setting the local dipole prefactor. The near-field source is therefore the transition-polarization density $\mathbf{P}_X(\mathbf{R}) = \mathbf{u} P_X(\mathbf{R})$ with $P_X(\mathbf{R}) \propto \psi_X(\mathbf{R})$, whose in-plane Fourier transform is $P_X(\mathbf{q}) = D_X F_X(q; \ell_X)$ with the normalized form factor

$$F_X(q; \ell_X) = \frac{\int d^2 R \psi_X(\mathbf{R}) e^{-i\mathbf{q} \cdot \mathbf{R}}}{\int d^2 R \psi_X(\mathbf{R})}. \quad (\text{B1})$$

The exciton center of mass sits in the moiré potential, which is harmonic near a minimum. Taking the origin at that minimum, $\mathbf{R}$ is the center-of-mass displacement from it and $V \simeq \frac{1}{2} M \omega_0^2 R^2$ with $R = |\mathbf{R}|$; its ground state is the normalized 2D Gaussian envelope $\psi_X(\mathbf{R}) = (\sqrt{\pi} \ell_X)^{-1} e^{-R^2/2\ell_X^2}$ ($\int d^2 R |\psi_X|^2 = 1$), with $\ell_X = \sqrt{\hbar/M\omega_0}$ the oscillator length set by the moiré curvature (M the exciton mass, ω_0 the trap frequency). This gives $F_X(q; \ell_X) = e^{-q^2 \ell_X^2/2}$, so the rate carries $|F_X(q; \ell_X)|^2 = e^{-q^2 \ell_X^2}$, Eq. (4). The total transition dipole is $D_X = \int d^2 R P_X(\mathbf{R}) \propto \int d^2 R \psi_X(\mathbf{R}) \propto \ell_X$ (all elements share the direction $\mathbf{u}$), i.e. $|D_X|^2 \propto \ell_X^2$. A coherent exciton delocalized over the area ℓ_X^2 carries a giant oscillator strength.

A different envelope changes $F_X(q; \ell_X)$ only at $q \gtrsim 1/\ell_X$ and does not alter the conclusion that the distance dependence probes the finite exciton size, quantified in general by the radius R_X of Sec. II C.

For comparison, the fixed-total-dipole benchmark treats the emitter as a rigid dipole $D_X = \text{const}$ spread with a normalized Gaussian transition-polarization profile $P_X(\mathbf{R}) \propto e^{-R^2/\ell_X^2}$, whose width coincides numerically with that of $|\psi_X(\mathbf{R})|^2$ for this Gaussian ψ_X but is not interpreted as a probability density. Its Fourier transform is $F_X(q; \ell_X) = e^{-q^2 \ell_X^2/4}$, so $|F_X(q; \ell_X)|^2 = e^{-q^2 \ell_X^2/2}$. This is a phenomenological rigid-dipole distribution rather than a coherent quantum transition polarization. It mimics the loss of coherent-area enhancement by holding the total dipole fixed, and isolates pure momentum filtering without the ℓ_X^2 prefactor. It recovers the point-dipole z^{-4} law for every ℓ_X at large z .

Appendix C: Practical aspects of the extraction protocol

The differential extraction protocol of Sec. IV is sensitivity- and statistics-limited in ways that set the achievable uncertainty on R_X .

Sensitivity and parameter correlations. The extraction is sensitivity-limited at large z . Since $|\partial \ln \Gamma_{\text{ET}} / \partial R_X| \simeq 5R_X/z^2$ is 0.15, 0.067, and 0.038 nm⁻¹ at $z = 10, 15$, and 20 nm for $R_X = 3$ nm, resolving R_X to ~ 0.5 nm near $z = 15$ nm requires measuring the rate to a few percent, exactly where Γ_{ET} is smallest. Moreover R_X correlates with the offset z_0 , the anisotropy γ , D_X , the fourth moment $\langle R^4 \rangle_P$, and the blocked-state residual. Turning these correlations into a realistic uncertainty on R_X then requires a joint (Fisher or profile-likelihood) fit of a multi-thickness series spanning both the crossover ($z \sim R_X$) and the asymptotic ($z \gg R_X$) regions.

Spacer offset. The distance z is measured from the exciton transition-polarization centroid. At nanometre spacings the finite TMDC-layer thickness, the interlayer-exciton position, and monolayer steps in the spacer all shift it and enter the z^{-4} scaling. The physical distance is $z_{\text{eff}} = \gamma(z_{\text{hBN}} + z_0)$ with an offset z_0 , and since $\delta \Gamma_{\text{ET}} / \Gamma_{\text{ET}} \simeq -4 \delta z / z$, a 0.2 nm error at $z = 3$ nm shifts the rate by $\sim 27\%$, comparable to the finite-size signal. The offset z_0 should therefore be fit as a nuisance parameter or calibrated independently, since it correlates strongly with R_X .

Device-to-device statistics. The ratio removes D_X only if the same exciton state and oscillator strength are maintained across the spacer-thickness series. Because a thickness series generally means different devices—with possible variation in twist angle, registry, strain, and disorder—the condition is best approached with a laterally stepped hBN spacer on a single heterobilayer, or by calibrating the oscillator strength for each device independently. The gate subtraction removes the intra-device baseline but not this device-to-device variation of D_X . For a laterally stepped spacer, exciton diffusion can also mix regions of different z if the diffusion length approaches the terrace width or the spot size, because strongly quenched regions act as sinks. Each terrace should therefore exceed the diffusion length, and the measurement be taken away from step edges and at low excitation density.

Because a given localized site cannot be placed at several spacer thicknesses, the comparison is across statistically matched lines, with $R_{X,i} = R_X + \delta R_i$ and $D_{X,i} = D_X + \delta D_i$ varying from line to line. Measuring several lines per thickness and fitting them as a random-effects

ensemble separates the mean R_X from this scatter.

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